# REPORT #3: ESTIMATION OF MAXIMAL ICU BEDS DEMAND FOR COVID-19 OUTBREAK IN SANTIAGO (CHILE) AND THE EFFECTS OF DIFFERENT MITIGATION STRATEGIES

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Abstract. In this document we use the compartmental epidemiological model introduced in Report #2 [3], in order to estimate the maximal ICU (intensive care unit) beds capacity required by a city (Santiago, Chile) during the COVID-19 outbreak, under the action of three classes of strategies. The maximal demand of ICU beds is presented as an output of our model, for lockdown strategy, the strategy consisting in contact tracing and isolation, and a combination of both mitigation measures.

Contents



**Disclaimer:** This report has been written under the urgency due to the current COVID-19 outbreak situation in Chile. It aims to present some mathematical modeling tools and their corresponding predictions, helping to justify important decisions by policymakers. This material will surely improve during next days, with the addition of more data and corresponding scientific exchanges with colleagues. In this regard, some projections inferred by this report may contain inaccuracies related to the unknown scientific aspects of the newly born disease. See all reports by our team at the webpage http://www.cmm.uchile.cl/covid-19-en-chile/ or http://matematica.usm.cl/covid-19-en-chile/.

## 1. Model and description of contention strategies

1.1. **Short description of the model.** The disease spread within a particular contaminated city (e.g. Santiago, Chile) has been modeled using a deterministic compartmental model (see, for instance [2] and references therein). In Reports 1 and 2 written previously [4, 3], our team has implemented this approach to the COVID-19 outbreak in Chile. The deterministic and compartmental approach that we develop here has some important advantages with respect to other approaches: among them, the most important are the simplicity and the rapidity to obtain results that can provide key insights and data for being used later in more complex models (e.g., stochastic, with interconnection between cities/districts, etc.).

The model proposed (see Appendix A for more details) consists in a compartmental model, where the population is distributed into 8 groups corresponding to different stages of the disease:

- **Susceptible** (denoted by $S$): Persons not infected by the disease, but able to be infected by the virus.
- **Exposed** (denoted by $E$): Persons in the incubation period after being infected by the disease. In this stage, persons **do not have symptoms but they can infect other people** with a lower probability than people in the infectious compartments described below.
- **Mild infected or subclinical** (denoted by $I^m$): Persons infected that can infect other people. Persons in this stage are asymptomatic or present mild symptoms, **they are not detected and then not reported by authorities**. At the end of this stage, they pass directly to recovered state.
- **Infected** (denoted by $I$): Persons infected that can infect other people. Persons in this stage develop symptoms and **are detected and then reported by authorities**. People in this stage can recover or pass to some hospitalized state.
- **Recovered** (denoted by $R$): People that survive the disease, **is no longer infectious and have developed immunity to the disease**.
- **Hospitalized** (denoted by $H$): Persons hospitalized in basic facilities. People in this stage can infect other people. **After this stage, people recover or pass to use a ICU bed**.
- **Hospitalized in ICU beds** (denoted by $H^c$): People hospitalized in ICU beds. People in this stage can infect other people. After this stage, **people die or are hospitalized in basic facilities**.
- **Dead** (denoted by $D$): People who did not survive the disease.

The choice of the above stages and the transition between them (described below) are because our main purpose is to estimate the **maximal demand of ICU beds**. For this reason we are modeling that all people that need a ICU bed will pass by stage $H^c$ without any constraint of availability.

As usual, these groups of stages are called state variables, so the vector of state variables is $\mathbf{x} = (S, E, I^m, I, R, H, H^c, D)$.

1.2. **Description of contention strategies.** The (indirect) control variables to be considered in our reports are the rate of contacts with infectious people. For a given time $t$ (measured in days), we denote by $u_X(t)$ the rate of contact of susceptible people with a person in the stage $X \in \{E, I^m, I, H, H^c\}$ at time $t \geq t_0$ ($t_0$ the considered initial time).

The rates of contagious at time $t \geq t_0$ are given by

$$\beta_X(t) = p_X u_X(t) \qquad t \geq t_0,\ X \in \{E, I^m, I, H, H^c\}, \tag{1}$$

where $p_X$ is the probability of a susceptible person ($S$) to be infected (i.e., to enter to the incubation stage $E$) after a contact with a person in the stage $X \in \{E, I^m, I, H, H^c\}$.

Additionally, for each control strategy $u_X(\cdot)$, with $X \in \{E, I^m, I\}$, we consider reference values (to be calibrated) $u_X^{\text{ref}} > 0$. If no mitigation strategy is applied in an interval of time $[t_1, t_2]$, one has

$$u_X(t) = u_X^{\text{ref}} \qquad \text{for all } \ t \in [t_1, t_2],\ X \in \{E, I^m, I\}. \tag{2}$$

Hence, mitigation strategies focused in reducing the contact rates satisfy $u_X(t) \in [0, u_X^{\text{ref}}]$ for all $t \geq t_0$, with $X \in \{E, I^m, I\}$. In this report we only consider strategies such that $u_X(t) = \alpha_X u_X^{\text{ref}}$, with $\alpha_X \in [0, 1]$. This value explains how the analyzed strategy impacts in the stage $X$.

Notice that, due to recommendations and most likely behavior, one should have

$$u_E^{\text{ref}} \approx u_{I^m}^{\text{ref}} > u_I^{\text{ref}} > u_H^{\text{ref}} \approx u_{H^c}^{\text{ref}} \approx 0, \tag{3}$$

because the contacts with people in incubation ($E$) or with mild symptoms ($I^m$) should be more frequent (because they do not know they are infected) than the contacts with infectious people with symptoms ($I$) or hospitalized ($H$ or $H^c$), and we assume that people hospitalized are highly isolated. Actually, from now on, we assume that $u_H^{\text{ref}} = u_{H^c}^{\text{ref}} = 0$. For this reason, $u_H$ and $u_{H^c}$ are not longer considered as control variables. This approach is also used in [11]. We summarize our assumptions on reference values $u_X^{\text{ref}}$ here below:

**Assumption 1.** *We assume the following on parameters $u_X^{\text{ref}}$: $u_E^{\text{ref}}$, $u_{I^m}^{\text{ref}}$, and $u_I^{\text{ref}}$:*

*(i)* $u_H^{\text{ref}} = u_{H^c}^{\text{ref}} = 0$, *because we assume that people hospitalized are highly isolated.*

*(ii)* $u_E^{\text{ref}} = u_{I^m}^{\text{ref}}$. *This means that people in the incubation stage have the same rate of contact than infected people with mild symptoms (and then, not detected).*

*(iii)* $\delta u_{I^m}^{\text{ref}} = u_I^{\text{ref}}$ *where* $\delta \in (0,1)$. *This represents that (symptomatic) infected people are more isolated than infected people with mild symptoms, unless an active search for subclinical cases would be implemented, strategy not considered in this report.*

We recall that the values $u_X^{\text{ref}}$ are obtained after a calibration procedure (see Appendix B). However, thanks to Assumption 1, now we only need to determine $u_E^{\text{ref}}$.

**Main objective of this report**

The main objective of this document is to report, for different strategies represented by $u_E(\cdot)$, $u_{I^m}(\cdot)$, and $u_I(\cdot)$, the maximal ICU beds demand. The mitigation measures to be considered in this report are lockdown strategy and the strategy consisting in the contact tracing and isolation.

1.2.1. *Lockdown and semi-lockdown strategies.* The application of a lockdown strategy in an interval of time $[t_{\text{ref}}, t_{\text{ref}} + T_{\text{L}}]$ is represented as a new control $u_X(t) = \alpha_{\text{L}} u_X^{\text{ref}}$ for all $t \in [t_{\text{ref}}, t_{\text{ref}} + T_{\text{L}}]$, for some particular $\alpha_{\text{L}} \in (0,1)$ representing the adopted measure. This choice is standard in the literature [7]. That is, we model a reduction of the contact rate during an interval of time. Thus, the control path $u_X(\cdot)$ is a piecewise constant function. We assume that the factor $\alpha_{\text{L}}$ is the same for the three controls $u_E(\cdot)$, $u_{I^m}(\cdot)$ and $u_I(\cdot)$. This choice is supported by the fact that in average, population is equally affected by this restriction measure, in our model and in reality. Other interesting option could be to consider non constant reduction of contact rates in a given interval, as in [10], trying to represent the adaptation of the population to the lockdown measure. We plan to use of this kind of representation in future reports.

We introduce the notation $\alpha_{\text{sL}} > \alpha_{\text{L}}$ for representing the reduction of contact rates due to a semi-lockdown strategy (lockdown of some districts of the city), as the current situation in Santiago.

We are aware that to represent lockdown and semi-lockdown strategies through a constant reduction of (average) contact rates is a somehow crude simplification. Nevertheless, for policymakers, we think that this approach is more illustrative and flexible than the representation of mitigation measures through reductions of the basic reproductive number (as we did in Report # 2), and also it is in line with recent literature and its recommendations [7, 8, 9, 10].

The contact rates $u_X(\cdot)$ for $X \in \{E, I^m, I\}$ during time, under lockdown strategy, is represented in Figure 1. In this figure, we are representing the semi-lockdown imposed at March 27. Thus, we will evaluate the measure of lockdown from a future date $t_{\text{ref}}$ during a period $T_{\text{L}}$, after which the lockdown will be lifted.

**Rate of contact during time for all contagious stages ($X \in \{E, I^m, I\}$)**

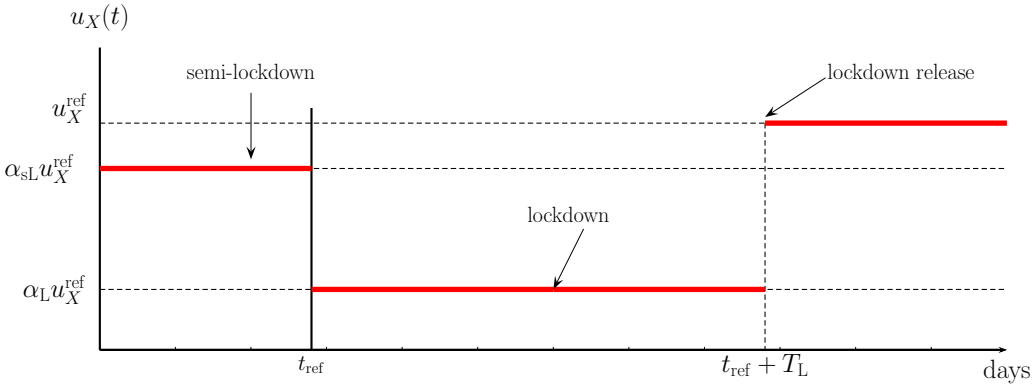


FIGURE 1. Rates of contacts $u_X(t)$ (for $X \in \{E, I^m, I\}$) during time representing lockdown strategy during a period $T_{\text{L}}$ starting in a future day $t_{\text{ref}}$.

1.2.2. *Contact tracing and isolation strategy (cti for short).* This strategy consists of increasing efforts to locate contacts of detected cases (for example, family, work and social contacts) and, subsequently, isolate these people (monitored quarantine). Very probably these individuals are already infected but perhaps they are in the incubation stage ($E$) or they will present mild symptoms ($I^m$). Therefore, the objective is to reduce the rate of contacts for people in compartments $E$ and $I^m$. Hence, we model this strategy by only reducing $u_E(t)$ and $u_{I^m}(t)$ during a long period of time. We think that this adequately represents the continued effort made in contact tracing and isolation, measure recommended by the Comité Asesor COVID-19 Chile, the advisor committee to the Chilean government, in its last meeting [1], where they propose to define the *probable case* state, that is, the contacts of detected people.

Given a future date $t_{\text{ref}}$ for starting the strategy, to be applied during a period of time $T_{\text{cti}}$, we represent the mitigation measure by $u_X(t) = \alpha_{\text{cti}} u_X^{\text{ref}}$ for $t \in [t_{\text{ref}}, t_{\text{ref}} + T_{\text{cti}}]$, where $\alpha_{\text{cti}} \in [0, 1)$, for $X \in \{E, I^m\}$, that is, the reduction of contact rates for exposed and infected people with mild symptoms. This strategy is depicted in Figure 2.

**Rate of contact during time for exposed and people with mild symptoms ($X \in \{E, I^m\}$)**

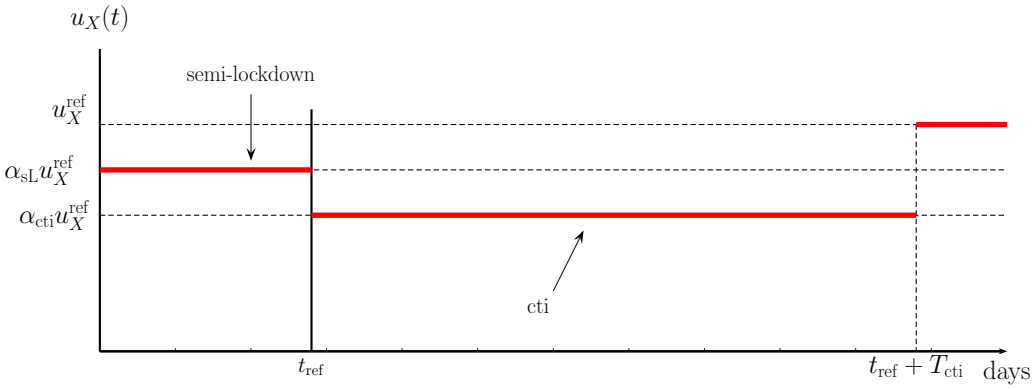


FIGURE 2. Rates of contacts $u_X(t)$ (for $X \in \{E, I^m\}$) during time representing contact tracing and isolation strategy during a period $T_{\text{cti}}$ starting in a future day $t_{\text{ref}}$.

| Notation | Value(s) | Meaning |
|---|---|---|
| $\delta$ | 0.2 | an symptomatic has $\delta$ times the contact rate of an infected with mild symptoms |
| $t_{\text{ref}}$ | April 6, 2020 | start of strategies |
| $T_{\text{L}}$ | 2 weeks | periods of lockdown procedure |
| $T_{\text{cti}}$ | 12 months | period for tracing contacts and isolation |
| $\alpha_{\text{L}}$ | 0.25 | reduction of contacts during lockdown |
| $\alpha_{\text{cti}}$ | 0.75, 0.5 | reduction of contacts during focalized quarantines (after tracing contacts) |
| $\alpha_{\text{sL}}$ | 0.8 | reduction of contacts during semi-lockdown procedure (now in Santiago) |

TABLE 1. Values used for the simulations of introduced strategies.

As can be seen in Figures 1 and 2 the introduced strategies look very similar. The main difference is that the lockdown strategy is applied to all population, while cti strategy is applied only to a subgroup of the population, namely, contacts traced of detected cases.

For the sake of simplicity, and also motivated by recent literature (e.g., [7, 8, 9]), we consider the values indicated in Table 1, for the parameters introduced in this section.

1.2.3. *Combination of strategies.* Finally, we consider a third strategy consisting in a combination of lockdown and cti strategy. Given a future date $t_{\text{ref}}$ for starting this strategy, we take two periods of time $T_{\text{L}} < T_{\text{cti}}$. In the interval $[t_{\text{ref}}, t_{\text{ref}} + T_{\text{L}}]$ the strategy lockdown is applied combining with the cti strategy, that is, for all $t \in [t_{\text{ref}}, t_{\text{ref}} + T_{\text{L}}]$ one has

$$u_X(t) = \alpha_{\text{L}}\ \alpha_{\text{cti}} u_X^{\text{ref}} \quad \text{for} \quad X \in \{E, I^m\};\ \text{and}\ \ u_I(t) = \alpha_{\text{L}} u_I^{\text{ref}}.$$

In addition, for all $t \in [t_{\text{ref}} + T_{\text{L}}, t_{\text{ref}} + T_{\text{cti}}]$ one has

$$u_X(t) = \alpha_{\text{cti}} u_X^{\text{ref}} \quad \text{for} \quad X \in \{E, I^m\}.$$

## 2. New results

We have simulated the model described in Report # 2 (see [3] or Appendix A), under the previous regimes, in the particular setting of Santiago, Chile. Our starting point has been the calibration of the data to match the observation of detected cases in Santiago [6] and the current *effective reproductive number* $\mathcal{R} = 1.75$ estimated by specialists and published in the press. This number has decreased drastically last days because of recent measures adopted by authorities, as closure of schools and universities [1], and the declared **semi-lockdown on March 27** to seven districts in Santiago (Independencia, Las Condes, Lo Barnechea, Ñuñoa, Providencia, Santiago and Vitacura; 1.3M people approx. and 23% of Santiago's population)[2].

Once this calibration is made, in a second step we performed several computational processes aiming to describe the behavior of the outbreak under the lockdown strategy described in Figure 1 and cti strategy depicted in Figure 2.

Then, given the parameters describen in Table 1 and Table 4 (in Appendix B) we run the model for the following strategies:

- **Baseline**: The semi-lockdown is released on April 6, 2020.
- **Strategy 1**: Lockdown (2 weeks from April 6, 2020);
- **Strategy 2a CTI (moderate)**: Contact tracing and isolation of detected cases with moderate intensity; Reduction of 25% of rates of contacts of people in incubation stage or infected with mild symptoms; (i.e., $\alpha_{\text{cti}} = 0.75$);
- **Strategy 2b CTI (high)**: Contact tracing and isolation of detected cases with high intensity; Reduction of 50% of rates of contacts of people in incubation stage or infected with mild symptoms; (i.e., $\alpha_{\text{cti}} = 0.5$).
- **Strategy 3**: Combination of Strategy 1 and Strategy 2b.

In Table 3 we report, associated to each strategy, the following indicators: the case fatality ratio (% of death over all infected detected cases), the maximal demand of hospitalized in non complex services ($H_{\max}$), the maximal demand of ICU beds ($H^c_{\max}$) and the dates when these two demands are reached (denoted by $t_{\max}$ and $t^c_{\max}$, respectively).

The evolution in time of hospitalized and the occupancy of UCI beds are depicted in Figures 3, 4, 5, and 6. In these figures, the strategies are compared with the a scenario without intervention, that is, current semi-lockdown regime is lifted on April 6.

[1] See https://www.minsal.cl/presidente-anuncia-suspension-de-clases-y-reduce-actos-publicos/ (in Spanish).

[2] See https://www.minsal.cl/ministro-de-salud-anuncio-cuarentena-total-para-siete-comunas-de-la-rm/ (in Spanish).

| **Strategy** | **Case fatality rate** | $H_{\max}$ | $t_{\max}$ (date) | $H^c_{\max}$ | $t^c_{\max}$ (date) |
|---|---|---|---|---|---|
| Baseline | 0.24% | 29407 | August 2 | 6962 | August 12 |
| 1 | 0.24% | 23160 | September 14 | 5554 | September 23 |
| 2a | 0.23% | 17023 | September 14 | 4133 | September 24 |
| 2b | 0.24% | 7493 | November 11 | 1851 | December 12 |
| 3 | 0.24% | 7415 | January 5 | 1831 | January 15 |

TABLE 2. Results obtained for strategies 1, 2a, 2b, and 3.

**Strategy 1: Lockdown (2 weeks)**

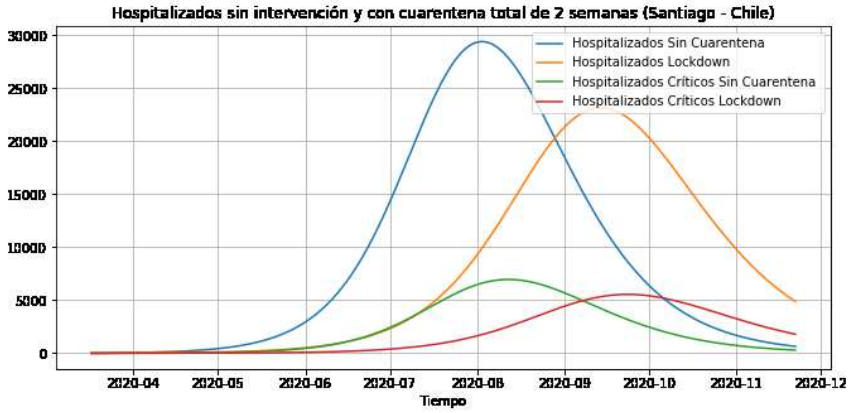


FIGURE 3. Lockdown strategy applied during 2 weeks since Monday April 6; Case fatality ratio = 0.24%.

**Strategy 2a: Contact tracing and isolation (moderate intensity)**

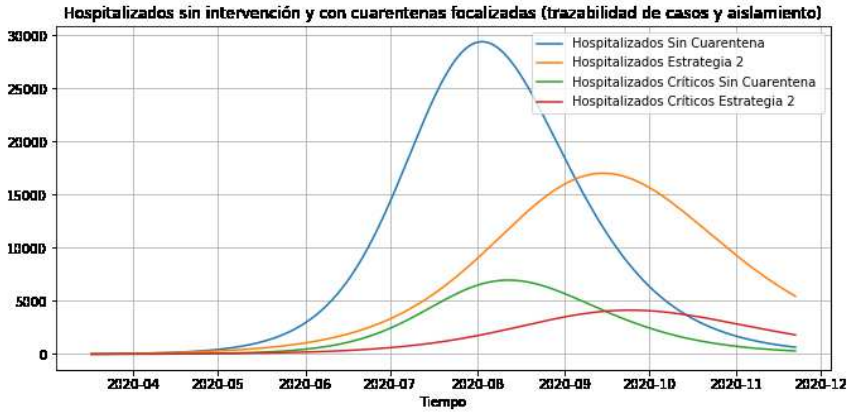


FIGURE 4. Cti strategy applied since Monday April 6 with moderate intensity ($\alpha_{\text{cti}} = 0.75$); Case fatality ratio = 0.23%.

**Strategy 2b: Contact tracing and isolation (high intensity)**

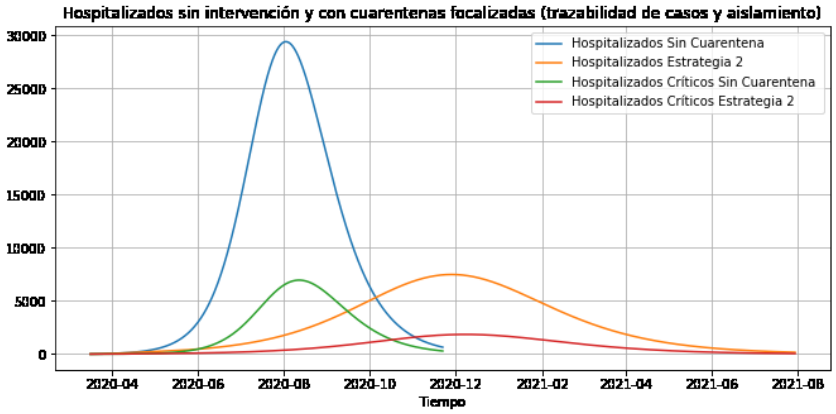


FIGURE 5. Cti strategy applied since Monday April 6 with high intensity ($\alpha_{\text{cti}} = 0.5$); Case fatality ratio = 0.24%.

**Strategy 3: Combined strategy**

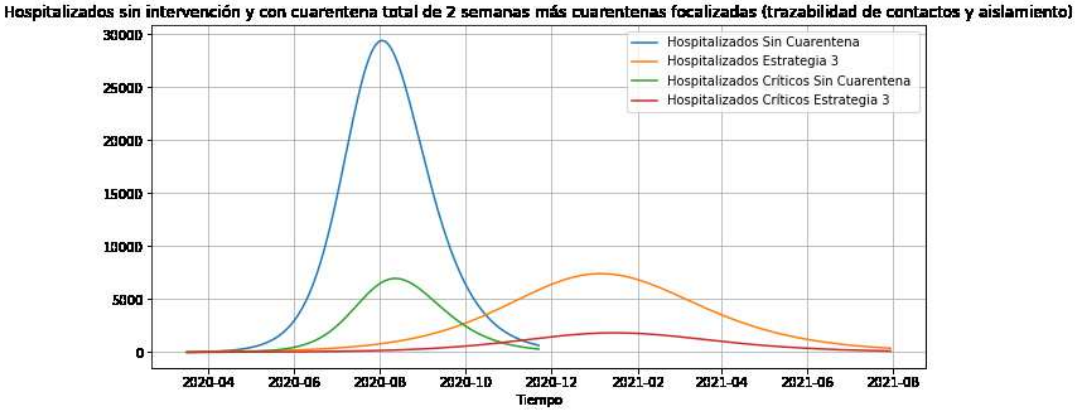


FIGURE 6. Strategy 3 applied since Monday April 6 with high intensity ($\alpha_{\text{cti}} = 0.5$); Case fatality ratio = 0.24%.

Our results describe **a shift in the peak with a small reduction in the amplitud in the demand for hospital resources** when lockdown strategy is implemented and a **flatten effect** of these curves when cti strategy is applied. The third strategy shows a combined effect.

## 3. Final remarks

- It is important to mention that there is among the scientific community great discrepancies on the exact percentage of asymptomatic/symptomatic persons present in this outbreak. Some international reports place the range between 20% and 50%. In previous report # 2, we informed three different scenarios, including few (20%), half (50%) and large (75%) amount of undetected contagious people. Here, in this report we inform our results with a 50% of ratio asymptomatic/symptomatic.

- Our model does not consider a learning effect of the population due to the application of lockdowns after lifting. Indeed, epidemiologists have pointed out to us that previous pandemics in Chile have strongly changed the behavior of the entire population for at least a long time. This idea will be explored in our future reports.

- Our model does not consider important consequences in the dynamics and health population due to the related economic crisis triggered by the COVID-19 outbreak. This phenomenon is of independent interest, and may be considered in forthcoming reports.

- Simulations for other cities, countries or regions can be easily implemented. In these moments, we are using data from China, reported in [10], to calibrate and test our model in a dataset more complete than what is available now in Chile.

- In a forthcoming report, we expect to describe our previous results applied this time to predict the future development of the outbreak along different regions in Chile.

- The parameters identification described in Appendix B is a very poor and ill-conditioned method. We are working on improving that. It is known (see [10]) that the parameter identification of an outbreak model before the peak can produce large errors in the outputs. For this reason, the approach introduced in this reports only allows estimating the magnitude order of maximal demands, but it is not appropriate for deducing an accurate estimation of daily cases.

- Despite all limitations above mentioned, we think the current development of the model might be useful to observe the direction of changes associated with different strategies. In this sense, to implement a total lockdown during two weeks and a tracing contact program with high intensity, isolating (with surveillance) the contacts of detected cases, shows qualitatively to be the best strategy.

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## Appendix A. Description of the model dynamics

In this Section we present additional information on the model that we consider in our simulations. For more details, the reader can consult our Report # 2 [3].

Recall the state variables $\mathbf{x} = (S, E, I^m, I, R, H, H^c, D)$ introduced in Section 1. In our model, the evolution of state variables is described by the following system of ordinary differential equations:

$$
\left\{
\begin{array}{rcl}
\dot{S} &=& \mu_b N - S\overbrace{\left(\frac{\beta_E E + \beta_{I^m} I^m + \beta_I I + \beta_H H + \beta_{H^c} H^c}{N}\right)}^{\Lambda(\mathbf{x},u)\text{: rate of contagious}} - \mu_d S \\
\dot{E} &=& S\left(\frac{\beta_E E + \beta_{I^m} I^m + \beta_I I + \beta_H H + \beta_{H^c} H^c}{N}\right) - (\gamma_E + \mu_d)E \\
\dot{I^m} &=& (1-\phi_{EI})\gamma_E E - (\gamma_{I^m} + \mu_d) I^m \\
\dot{I} &=& \phi_{EI}\gamma_E E - (\gamma_I + \mu_d) I \\
\dot{R} &=& \gamma_{I^m} I^m + \phi_{IR}\gamma_I I + \phi_{HR}\gamma_H H - \mu_d R \\
\dot{H} &=& (1-\phi_{IR})\gamma_I I + (1-\phi_D)\gamma_{H^c} H^c - (\gamma_H + \mu_d) H \\
\dot{H^c} &=& (1-\phi_{HR})\gamma_H H - (\gamma_{H^c} + \mu_d) H^c \\
\dot{D} &=& \phi_D \gamma_{H^c} H^c.
\end{array}
\right. \tag{4}
$$

This model represents an extension of a SEIRHD model which aims to better describe an outbreak where part of the population has been infected by a virus, but an important part presents no or just mild symptoms. It turns out that this is the particular case of the virus SARS-CoV-2, as it is presented in several international reports [7, 10]. Schematically speaking, the structure of the model with the transitions between different stages is presented in Figure 7.

## Appendix B. Parameters and calibration

The parameters to be identified (literature and/or calibration) are

$$
P = (p, \mu_b, \mu_d, \gamma, \phi, u^{\text{ref}}) \in [0,1]^5 \times \mathbb{R}_+ \times \mathbb{R}_+ \times [0,1]^5 \times [0,1]^3 \times \mathbb{R}_+^5 \subset \mathbb{R}^{20}. \tag{5}
$$

The descriptions of these parameters are the following:

- $p = (p_E, p_{I^m}, p_I, p_H, p_{H^c})$ are the probabilities of contagious (see (1)) when a susceptible person is in contact with a person in stages $E$, $I^m$, $I$, $H$, and $H^c$.
- $\mu_b$ is the natality rate in the city and $\mu_d$ is the mortality rate, both measured in $[\text{day}]^{-1}$;[3]
- Parameters $\gamma_X$ measured in $[\text{day}]^{-1}$ are the rate of transition from a disease stage $X \in \{E, I^m, I, H, H^c\}$ to the following stage, where $\gamma_X^{-1}$ represents the mean duration of stage $X$;
- $\phi_{EI}$ is the fraction of exposed people who become infected (with symptoms);
- $\phi_{IR}$ is the fraction of infected people that recover;

[3]Note that our simulations are for a particualr period of time (less than one year), in which case these rates do not impose important changes to the population size.

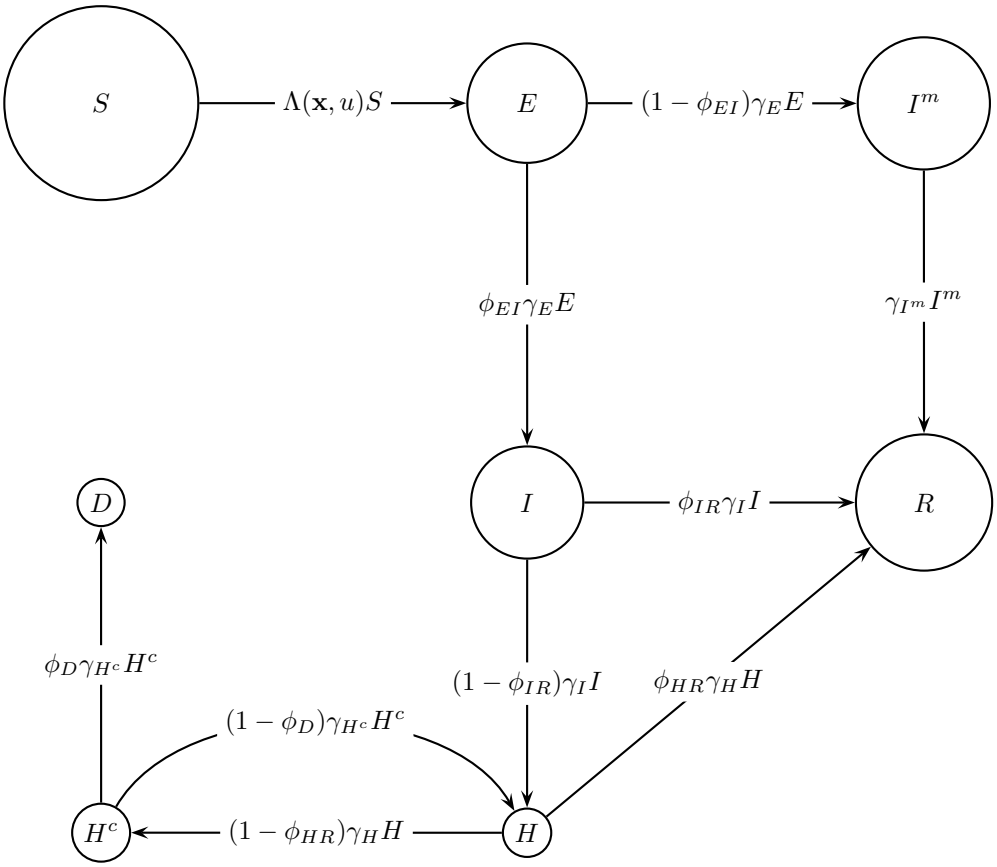


FIGURE 7. **Structure of the mathematical model for the dynamics of COVID-19 in an isolated city.** Each circle represents a compartment. Susceptible individuals ($S$), and different disease states: exposed ($E$), mild infected ($I^m$), infected ($I$), recovered ($R$), hospitalized ($H$), hospitalized in ICU beds ($H^c$), and dead ($D$). Natural natality and mortality flows are not represented.

- $\phi_{HR}$ is the fraction of hospitalized (in normal services) people that recover;
- $\phi_D$ is the fraction of hospitalized people in ICU beds that die;
- The vector $u^{\text{ref}} = (u_E^{\text{ref}}, u_{I^m}^{\text{ref}}, u_I^{\text{ref}}, u_H^{\text{ref}}, u_{H^c}^{\text{ref}})$ contains references values of rates of contact.

Unfortunately, we have not found yet literature about the probabilities of contagious $p_X$. The most used modeling approach in the recent literature related to COVID-19 is to estimate the rates of contagious $\beta_X$ (see (1)). We have preferred to separate the probability of contagious $p_X$ and the contact rates $u_X$ because these quantities are comparable between different stages of the desease while the contagious rates are not. In Assumption 1 we take as hypothesis some relations between contact rates. In the next assumption we proceed similarly with the probabilities of contagious, based in discussions with epidemiologists.

**Assumption 2.** *We assume the following on parameters $p_X$: $p_E$, $p_{I^m}$, $p_I$:*

*(i) $p_E = 0.5 p_{I^m}$, because in part of the exposed stage (first 2-3 days) people are not contagious;*

*(ii)* $p_{I^m} = 0.15 p_I$*, because people infected without or with mild symptoms are considerable less contagious than infected people with symptoms, for instance, they do not cough.*

Thanks to Assumption 2, in order to determine the probabilities of contagious, we only need to determine or fix the value $p_I$. In this report we use the value $p_I = 0.75$.

Based in the daily reports given in [6] we take the following values of ratios $\phi_{EI}$, $\phi_{IR}$, $\phi_{HR}$, and $\phi_D$.

| **Notation** | **Value** | **Meaning** |
|---|---|---|
| $\phi_{ER}$ | 0.85 | fraction of infected people (with symptoms and detected) that recover |
| $\phi_{HR}$ | 0.85 | fraction of hospitalized people that recover |
| $\phi_D$ | 0.1 | fraction of hospitalized people in UCI bed that die |

TABLE 3. Values of parameters $\phi_{EI}$, $\phi_{IR}$, $\phi_{HR}$ deduced from [6].

Recall that $\phi_{EI} \in [0, 1]$ is the fraction of exposed people that will present symptoms. These persons are identified and being passed to the infected compartment ($I$), and not to ($I^m$) (see system (4) or Figure 7). This fraction is a parameter but in this report we fix the value $\phi_{EI} = 0.5$ (see Report # 2 [3] for scenarios associated to this parameter).

For the natality and mortality rates we take the values estimated from CENSO 2017 Chile, that is $\mu_b = 3.57 \cdot 10^{-5}$ and $\mu_d = 1.57 \cdot 10^{-5}$ both measured in $[\text{day}]^{-1}$.

For the rest of parameters, we consider a range of values taken from literature and the consideration of the authors of this report.

| **Notation** | **Unit** | **Range of values** | **References** |
|---|---|---|---|
| $\gamma_E$ | $[\text{day}]^{-1}$ | $[1/6, 1/4]$ | [5, 10, 14] |
| $\gamma_{I^m}$ | $[\text{day}]^{-1}$ | $[1/14, 1/7]$ | [5, 10] |
| $\gamma_I$ | $[\text{day}]^{-1}$ | $[1/14, 1/7]$ | [10, 13] |
| $\gamma_H$ | $[\text{day}]^{-1}$ | $[1/10, 1/2]$ | [7, 11, 12] |
| $\gamma_{H^c}$ | $[\text{day}]^{-1}$ | $[1/16, 1/7]$ | [5, 7] |
| $u_E^{\text{ref}}$ | none | $[0, 0.8]$ | modeling team |

TABLE 4. Range of values for parameters used in model (4).

For a vector of parameters $P$ in the ranges given in Table 4, we compute the detected cases at day $d \in \{03/03, \ldots, \text{today}\}$ given by model (4), that is

$$C(d, P) = \int_{t_0}^{d} \phi_{EI} \gamma_E E(t) dt.$$

This allows selecting the unfixed parameters in $P$ to fit the above quantity to daily reports until today and also the current effective reproductive number $\mathcal{R} = 1.75$ estimated daily by specialists as Mauricio Canals (Escuela de Salud Pública, Universidad de Chile) and published in the press.

In Figure 8 we show the curve obtained by the above procedure in comparison with daily number of detected cases reported by authorities

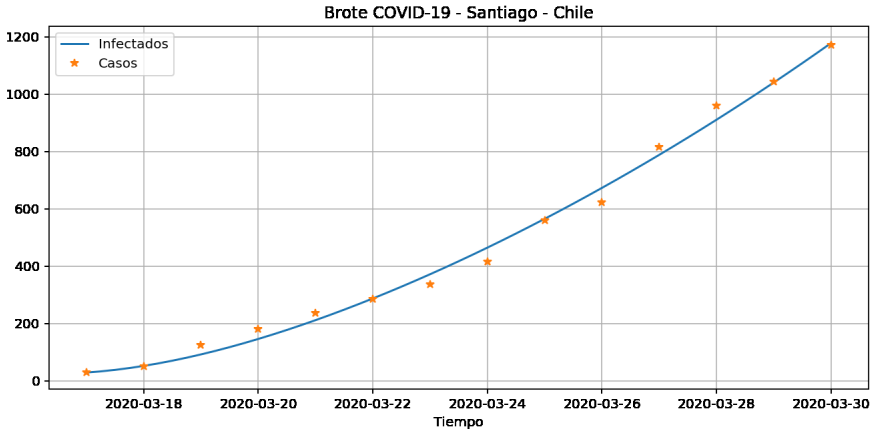


FIGURE 8. Detected infected cases: daily reports and model output for a effective reproductive number $\mathcal{R} = 1.75$.

This calibration process leads to the selection of the parameters of model (4) for Santiago's outbreak, reported in Table 5.

| Notation | Unit | Values used by our model | References |
|---|---|---|---|
| $p_E$ | none | 0.0563 | Assumption 2 |
| $p_{I^m}$ | none | 0.1125 | Assumption 2 |
| $p_I$ | none | 0.75 | Assumption 2 |
| $\mu_b$ | $[\text{day}]^{-1}$ | $3.57 \cdot 10^{-5}$ | INE-Chile (2017) |
| $\mu_d$ | $[\text{day}]^{-1}$ | $1.57 \cdot 10^{-5}$ | INE-Chile (2017) |
| $\gamma_E$ | $[\text{day}]^{-1}$ | 0.2 | [5, 10, 14], fitted |
| $\gamma_{I^m}$ | $[\text{day}]^{-1}$ | 0.1 | [5, 10], fitted |
| $\gamma_I$ | $[\text{day}]^{-1}$ | 0.1 | [10, 13], fitted |
| $\gamma_H$ | $[\text{day}]^{-1}$ | 0.1666 | [7, 11, 12], fitted |
| $\gamma_{H^c}$ | $[\text{day}]^{-1}$ | 0.1 | [5, 7], fitted |
| $\phi_{EI}$ | none | 0.5 | [6], modeling team |
| $\phi_{IR}$ | none | 0.85 | [6], modeling team |
| $\phi_{HR}$ | none | 0.85 | [7, 11, 6], modeling team |
| $\phi_D$ | none | 0.1 | [7, 10, 6], modeling team |
| $u_E^{\text{ref}}$ | none | 1.1025 | Fitted |
| $u_{I^m}^{\text{ref}}$ | none | 1.1025 | Assumption 1 |
| $u_I^{\text{ref}}$ | none | 0.2205 | Assumption 1 |
| $u_H^{\text{ref}}$ | none | 0 | Assumption 1 |
| $u_{H^c}^{\text{ref}}$ | none | 0 | Assumption 1 |

TABLE 5. Values for parameters used in model (4).

Finally, as part of the calibration process, we also fit initial values for exposed and mild infected persons ($E_0$ and $I_0^m$, respectively), at initial time $t_0$ = March 17, 2020. Recall that for the estimation of initial conditions we consider the total population in of Santiago (CENSO 2017), which is 5.624 millions people, and an estimation of cases until today. All these values used summarized below in Table 6.

| State | Value (individuals) | Source |
|---|---|---|
| $S_0$ | $5.624 \cdot 10^6$ | Censo 2017 |
| $E_0$ | 129 | Fitted |
| $I_0^m$ | 1799 | Fitted |
| $I_0$ | 174 | [6] |
| $H_0$ | 40 | modeling team |
| $H_0^c$ | 4 | modeling team |
| $R_0$ | 0 | modeling team |
| $D_0$ | 0 | [6] |

TABLE 6. Initial conditions for (4), considering the total population of Santiago, for initial time $t_0$ = March 17, 2020.

## APPENDIX C. COMPUTATION OF THE BASIC REPRODUCTIVE NUMBER $\mathcal{R}_0$

In this section we report the expression for the basic reproductive number $\mathcal{R}_0$ of model (4). For this, we first do the following approximations:

$$\Gamma_E := (\gamma_E+\mu_d) \approx \gamma_E; \quad \Gamma_I := (\gamma_I+\mu_d) \approx \gamma_I; \quad \Gamma_H := (\gamma_H+\mu_d) \approx \gamma_H; \quad \Gamma_{H^c} := (\gamma_{H^c}+\mu_d) \approx \gamma_{H^c}.$$

Then, for the sake of simplicity, we introduce the next notation:

$$\Psi_D := (1-\phi_D); \quad \Psi_{HR} := (1-\phi_{HR}); \quad \Psi_{IR} := (1-\phi_{IR})$$

We thus obtain the following expression

$$\begin{aligned} R0 = &\frac{\gamma_E\gamma_H\gamma_I p_{H^c} u_{H^c}\phi_{EI}\Psi_{HR}\Psi_{IR}}{(-\gamma_{H^c}\gamma_H\Gamma_E\Gamma_I\Psi_D\Psi_{HR}+\Gamma_{H^c}\Gamma_E\Gamma_H\Gamma_I)} + \frac{\gamma_E\phi_{EI}p_I u_I}{(\Gamma_E\Gamma_I)} + \frac{\gamma_E p_{I^m} u_{I^m}\Psi_{EI}}{(\Gamma_E\Gamma_{I^m})} \\ &+\frac{p_E u_E}{\Gamma_E} + \frac{p_H u_H\gamma_{H^c}\gamma_E\gamma_H\gamma_I\phi_{EI}\Gamma_E\Gamma_I\Psi_D\Psi_{HR}\Psi_{IR}}{(\Gamma_E\Gamma_H\Gamma_I(-\gamma_{H^c}\gamma_H\Gamma_E\Gamma_I\Psi_D\Psi_{HR}+\Gamma_{H^c}\Gamma_E\Gamma_H\Gamma_I))} \\ &+\frac{p_H u_H\gamma_E\gamma_I\phi_{EI}\Psi_{IR}(-\gamma_{H^c}\gamma_H\Gamma_E\Gamma_I\Psi_D\Psi_{HR}+\Gamma_{H^c}\Gamma_E\Gamma_H\Gamma_I)}{(\Gamma_E\Gamma_H\Gamma_I(-\gamma_{H^c}\gamma_H\Gamma_E\Gamma_I\Psi_D\Psi_{HR}+\Gamma_{H^c}\Gamma_E\Gamma_H\Gamma_I))}. \end{aligned}$$

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