

1 Syntax

$$\begin{array}{l}
 e ::= c \in \mathbb{Z} \\
 \quad | x \in Var \\
 \quad | e_1 + e_2
 \end{array}
 \quad
 \begin{array}{l}
 e_e ::= \cdot +_1 e \\
 \quad | \cdot +_2 \cdot
 \end{array}
 \quad
 \begin{array}{l}
 s ::= skip \\
 \quad | s_1; s_2 \\
 \quad | x := e \\
 \quad | if (e > 0) s_1 s_2 \\
 \quad | while (e > 0) s
 \end{array}
 \quad
 \begin{array}{l}
 s_e ::= x :=_1 \cdot \\
 \quad | \cdot;_1 s_2 \\
 \quad | if_1 s_1 s_2 \\
 \quad | while_1 (e > 0) s \\
 \quad | while_2 (e > 0) s
 \end{array}$$

2 Semantics

2.1 Expressions

$$\begin{array}{c}
 \frac{RED-CONST(c)}{E^\sharp, c \Downarrow \alpha(c)} \quad \frac{[y \quad red-var \quad rule : while : red : var E^\sharp, x \Downarrow E^\sharp[x] \mid_{Val^\sharp}]}{(x)}
 \end{array}$$

$$\begin{array}{c}
 \frac{RED-VAR-UNDEF(x)}{E^\sharp, x \Downarrow err^\sharp} \quad \frac{undef^\sharp \subset E^\sharp[x]}{E^\sharp, e_1 \Downarrow r^\sharp} \quad \frac{RED-ADD(e_1, e_2)}{E^\sharp, e_1 \Downarrow r^\sharp \quad E^\sharp, r^\sharp, \cdot +_1 e_2 \Downarrow r'^\sharp} \\
 \frac{RED-ADD-1(e_2)}{E^\sharp, e_2 \Downarrow r^\sharp \quad E^\sharp, r_1^\sharp \mid_{Val^\sharp}, r^\sharp, \cdot +_2 \cdot \Downarrow r'^\sharp} \\
 \frac{RED-ADD-2}{E^\sharp, v_1^\sharp, r_2^\sharp, \cdot +_2 \cdot \Downarrow v_1^\sharp +^\sharp r_2^\sharp \mid_{Val^\sharp}}
 \end{array}$$

2.2 Statements

$$\begin{array}{c}
\text{RED-SKIP} \\
\frac{}{E^\sharp, \text{skip} \Downarrow E^\sharp} \\
\\
\text{RED-SEQ}(s_1, s_2) \\
\frac{E^\sharp, s_1 \Downarrow r^\sharp \quad r^\sharp, \cdot;_1 s_2 \Downarrow r'^\sharp}{E^\sharp, s_1; s_2 \Downarrow r'^\sharp} \\
\\
\text{RED-SEQ-1}(s_2) \\
\frac{E^\sharp, s_2 \Downarrow r^\sharp}{E^\sharp, \cdot;_1 s_2 \Downarrow r^\sharp} \\
\\
\text{RED-ASN}(x, e) \\
\frac{E^\sharp, e \Downarrow r^\sharp \quad E^\sharp, r^\sharp, x :=_1 \cdot \Downarrow r'^\sharp}{E^\sharp, x := e \Downarrow r'^\sharp} \\
\\
\text{RED-ASN-1}(x) \\
\frac{}{E^\sharp, r^\sharp, x :=_1 \cdot \Downarrow E^\sharp [x \leftarrow r^\sharp \mid_{\text{Val}^\sharp}]} \\
\\
\text{RED-IF}(e, s_1, s_2) \\
\frac{E^\sharp, e \Downarrow r^\sharp \quad E^\sharp, r^\sharp, \text{if}_1 s_1 s_2 \Downarrow r'^\sharp}{E^\sharp, \text{if} (e > 0) s_1 s_2 \Downarrow r'^\sharp} \\
\\
\text{RED-IF-1-POS}(s_1, s_2) \\
\frac{E^\sharp, s_1 \Downarrow r'^\sharp}{E^\sharp, r^\sharp, \text{if}_1 s_1 s_2 \Downarrow r'^\sharp} \quad r^\sharp \mid_{\text{Val}^\sharp} \sqcap + \neq \perp \\
\\
\text{RED-IF-1-NEG}(s_1, s_2) \\
\frac{E^\sharp, s_2 \Downarrow r'^\sharp}{E^\sharp, r^\sharp, \text{if}_1 s_1 s_2 \Downarrow r'^\sharp} \quad r^\sharp \mid_{\text{Val}^\sharp} \sqcap - 0 \neq \perp \\
\\
\text{RED-WHILE}(e, s) \\
\frac{E^\sharp, e \Downarrow r^\sharp \quad E^\sharp, r^\sharp, \text{while}_1 (e > 0) s \Downarrow r'^\sharp}{E^\sharp, \text{while} (e > 0) s \Downarrow r'^\sharp} \\
\\
\text{RED-WHILE-1-NEG}(e, s) \\
\frac{}{E^\sharp, r^\sharp, \text{while}_1 (e > 0) s \Downarrow E^\sharp} \quad r^\sharp \mid_{\text{Val}^\sharp} \sqcap - 0 \neq \perp \\
\\
\text{RED-WHILE-1-POS}(e, s) \\
\frac{E^\sharp, s \Downarrow r'^\sharp \quad r'^\sharp, \text{while}_2 (e > 0) s \Downarrow r''^\sharp}{E^\sharp, r^\sharp, \text{while}_1 (e > 0) s \Downarrow r''^\sharp} \quad r^\sharp \mid_{\text{Val}^\sharp} \sqcap + \neq \perp \\
\\
\text{RED-WHILE-2}(e, s) \\
\frac{E^\sharp, \text{while} (e > 0) s \Downarrow r^\sharp}{E^\sharp, \text{while}_2 (e > 0) s \Downarrow r^\sharp}
\end{array}$$

2.3 Aborting Rules

$$\begin{array}{c}
\text{RED-ERROR-EXPR}(e) \\
\frac{}{\sigma^\sharp, e \Downarrow \text{err}^\sharp} \quad \text{abort } \sigma^\sharp \\
\\
\text{RED-ERROR-STAT}(s) \\
\frac{}{\sigma^\sharp, s \Downarrow \text{err}^\sharp} \quad \text{abort } \sigma^\sharp \\
\\
\frac{\sigma^\sharp = C[\text{err}^\sharp]}{\text{abort } \sigma^\sharp}
\end{array}$$