1 Syntax

$$
\begin{aligned}
e ::=\ & c \in \mathbb{Z} \\
\mid\ & \mathsf{x} \in \mathit{Var} \\
\mid\ & e_1 + e_2 \\
\mid\ & \lambda \mathsf{x}.s \\
\mid\ & e_1\,(e_2)
\end{aligned}
\qquad
\begin{aligned}
e_e ::=\ & \cdot +_1 e \\
\mid\ & \cdot +_2 \cdot \\
\mid\ & @_1\,(e_2) \\
\mid\ & @_2 \\
\mid\ & @_3
\end{aligned}
\qquad
\begin{aligned}
s \in \mathit{stat} ::=\ & \mathit{skip} \\
\mid\ & s_1 ; s_2 \\
\mid\ & \mathsf{x} := e \\
\mid\ & \mathit{if}\ (e > 0)\ s_1\ s_2 \\
\mid\ & \mathit{while}\ (e > 0)\ s \\
\mid\ & \mathit{return}\, e
\end{aligned}
\qquad
\begin{aligned}
s_e ::=\ & \mathsf{x} :=_1 \cdot \\
\mid\ & \cdot ;_1 s_2 \\
\mid\ & \mathit{if}_1\ s_1\ s_2 \\
\mid\ & \mathit{while}_1\ (e > 0)\ s \\
\mid\ & \mathit{while}_2\ (e > 0)\ s \\
\mid\ & \mathit{return}_1 \cdot
\end{aligned}
$$

2 Semantics

2.1 Expressions

$$\textsc{red-const}(c)\quad \frac{}{H_e, \ell_e, \ell_c, c \Downarrow H_e, \ell_e, c}$$

$$\textsc{red-var-local}(\mathsf{x})\quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, \ell_c\,[\mathsf{x}]}\quad \mathsf{x} \in \mathit{dom}\,(H_e\,[\ell_c])$$

$$\textsc{red-var-global}(x)\quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow H_e, \ell_e, E\,[\mathsf{x}]}\quad \mathsf{x} \in \mathit{dom}\,(H_e\,[\ell_e]) \wedge \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_c])$$

$$\textsc{red-var-undef}(x)\quad \frac{}{H_e, \ell_e, \ell_c, \mathsf{x} \Downarrow \mathit{err}}\quad \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_e]) \wedge \mathsf{x} \notin \mathit{dom}\,(H_e\,[\ell_c])$$

$$\textsc{red-add}(e_1, e_2)\quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, \cdot +_1 e_2 \Downarrow r'}{H_e, \ell_e, \ell_c, e_1 + e_2 \Downarrow r'}$$

$$\textsc{red-add-1}(e_2)\quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad v_1, r, \cdot +_2 \cdot \Downarrow r'}{\ell_c, (H_e, \ell_e, v_1), \cdot +_1 e_2 \Downarrow r'}$$

$$\textsc{red-add-2}\quad \frac{}{v_1, (H_e, \ell_e, v_2), \cdot +_2 \cdot \Downarrow H_e, \ell_e, v_1 + v_2}$$

$$\textsc{red-lambda}(\mathsf{x}, s)\quad \frac{}{H_e, \ell_e, \ell_c, \lambda \mathsf{x}.s \Downarrow H_e, \ell_e, (\ell_c, \lambda \mathsf{x}.s)}$$

$$\textsc{red-app}(e_1, e_2)\quad \frac{H_e, \ell_e, \ell_c, e_1 \Downarrow r \qquad \ell_c, r, @_1\,(e_2) \Downarrow r'}{H_e, \ell_e, \ell_c, e_1\,(e_2) \Downarrow r'}$$

$$\textsc{red-app-1}(e_2)\quad \frac{H_e, \ell_e, \ell_c, e_2 \Downarrow r \qquad \ell_c', \mathsf{x}, s, r, @_2 \Downarrow r'}{\ell_c, (H_e, \ell_e, (\ell_c', \lambda \mathsf{x}.s)), @_1\,(e_2) \Downarrow r'}$$

$$\textsc{red-app-2}(s)\quad \frac{\ell_c' = \mathit{fresh}\,(H_e) \qquad C = H_e\,[\ell_c] \\ H_e\,[\ell_c' \leftarrow C\,[\mathsf{x} \leftarrow v]], \ell_e, \ell_c', s \Downarrow r \qquad r, @_3 \Downarrow r'}{\ell_c, \mathsf{x}, s, (H_e, \ell_e, v), @_2 \Downarrow r'}$$

$$\textsc{red-app-3-ret}\quad \frac{}{\mathit{ret}\,(H_e, \ell_e, v), @_3 \Downarrow H_e, \ell_e, v}$$

$$\textsc{red-app-3-no-ret}\quad \frac{}{H_e, \ell_e, \ell_c, @_3 \Downarrow \mathit{err}}$$

2.2 Statements

RED-SKIP

$$\frac{}{H_e, \ell_e, \ell_c, \mathit{skip} \Downarrow H_e, \ell_e, \ell_c}$$

RED-SEQ(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r \qquad r, \cdot ;_1 s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, s_1 ; s_2 \Downarrow r'}$$

RED-SEQ-1(s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{H_e, \ell_e, \ell_c, \cdot ;_1 s_2 \Downarrow r}$$

RED-ASN(x, e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathsf{x} :=_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathsf{x} := e \Downarrow r'}$$

RED-ASN-1(x)

$$\frac{\ell'_e = \mathit{fresh}\left(H_e\right) \qquad E = H_e\left[\ell_e\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell'_e \leftarrow E\left[\mathsf{x} \leftarrow v\right]\right], \ell'_e, \ell_c} \quad \mathsf{x} \notin \mathit{dom}\left(H_e\left[\ell_c\right]\right)$$

RED-ASN-1-LOCAL(x)

$$\frac{\ell'_c = \mathit{fresh}\left(H_e\right) \qquad C = H_e\left[\ell_c\right]}{\ell_c, \left(H_e, \ell_e, v\right), \mathsf{x} :=_1 \cdot \Downarrow H_e\left[\ell'_c \leftarrow C\left[\mathsf{x} \leftarrow v\right]\right], \ell_e, \ell'_c} \quad \mathsf{x} \in \mathit{dom}\left(C\right)$$

RED-IF(e, s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{if}_1\, s_1\, s_2 \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{if}\ \left(e > 0\right)\ s_1\, s_2 \Downarrow r'}$$

RED-IF-1-POS(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_1 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v > 0$$

RED-IF-1-NEG(s_1, s_2)

$$\frac{H_e, \ell_e, \ell_c, s_2 \Downarrow r}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{if}_1\, s_1\, s_2 \Downarrow r} \quad v \leqslant 0$$

RED-WHILE(e, s)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad \ell_c, r, \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r'}$$

RED-WHILE-1-NEG(e, s)

$$\frac{}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow H_e, \ell_e, \ell_c} \quad v \leqslant 0$$

RED-WHILE-1-POS(e, s)

$$\frac{H_e, \ell_e, \ell_c, s \Downarrow r \qquad r, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r'}{\ell_c, \left(H_e, \ell_e, v\right), \mathit{while}_1\ \left(e > 0\right)\ s \Downarrow r'} \quad v > 0$$

RED-WHILE-2(e, s)

$$\frac{H_e, \ell_e, \ell_c, \mathit{while}\ \left(e > 0\right)\ s \Downarrow r}{H_e, \ell_e, \ell_c, \mathit{while}_2\ \left(e > 0\right)\ s \Downarrow r}$$

RED-RETURN(e)

$$\frac{H_e, \ell_e, \ell_c, e \Downarrow r \qquad r, \mathit{return}_1 \cdot \Downarrow r'}{H_e, \ell_e, \ell_c, \mathit{return}\, e \Downarrow r'}$$

RED-RETURN-1

$$\frac{}{\left(H_e, \ell_e, v\right), \mathit{return}_1 \cdot \Downarrow \mathit{ret}\left(H_e, \ell_e, v\right)}$$

2.3 Aborting Rules

$$\frac{}{\sigma, e \Downarrow err}\ \textsc{red-error-expr}(e)\quad \mathbf{abort}\,\sigma \wedge \neg\mathbf{intercept}_e\,\sigma$$

$$\frac{}{\sigma, s \Downarrow err}\ \textsc{red-error-stat}(s)\quad \mathbf{abort}\,\sigma$$

$$\frac{\sigma = C\,[\,err\,]}{\mathbf{abort}\,\sigma}$$

$$\frac{}{\mathbf{intercept}_{@_3}\, ret\,(H_e, \ell_e, v)}$$