

**SEMINARIO**

**MECANICA MATEMATICA Y PROBLEMAS INVERSOS.**

**EXPOSITOR**

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**TITULO**

***Hierarchy of pre-strained behaviors for thin materials***

**ABSTRACT:**

A hierarchy of four models for thin structures was first obtained by means of a systematic, but formal, approach in [5]. Namely, the nonlinear membrane model, the nonlinear bending model, the slightly nonlinear von Kármán model and the linear model were recovered. Emphasis was put on the loading order of magnitude and on the induced magnitude of the deformations. A rigorous variational derivation of the nonlinear membrane model was then given in [7]. In particular, the degeneracy under compression was exhibited. Then, [6] provided a rigidity result that allowed to rigorously justify the other models.

Some recent papers by physicists [4] suggest models where thin bodies (leaves, gel disks) try to reach an elastic equilibrium state after some given non Euclidean metrics has been imposed and, possibly, fail when considered as 3d bodies (case when the 3d metric tensor has no realization); see also [3]. Which thin limit models should be used? It turns out, as shown in [2], [8], [9], that the driving mechanism is the structure of the 3d Riemann curvature tensor  $R$ . Depending on the vanishing of blocks of entries of  $R$ , models that share several features with the Euclidean hierarchy are derived. The method of proof requires extended quantitative rigidity estimates.

This is joint work with Marta Lewicka, University of Pittsburgh.

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Martes 05 de Enero del 2016 a las 16:00 hrs, Sala de Seminarios John Von Neumann CMM, séptimo piso, torre norte, Beauchef 851.

