

SEMINARIO

OPTIMIZATION Y EQUILIBRIO

EXPOSITOR

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TITULO

The Schur property over some Lipschitz-free spaces.

Abstract:

A very classical question in classification of Banach spaces is the following: Consider X and Y two separable Banach spaces which are Lipschitz isomorphic (i.e. there exist a bijective bi-Lipschitz map between X and Y), do X and Y are necessarily linearly isomorphic? In this context, Godefroy and Kalton had introduced Lipschitz-free spaces in order to explore related questions (2003). However, Lipschitz-free spaces were already studied before (for instance by Weaver who called them Arens-Eells spaces, 1999).

The definition of the Lipschitz-free space over a metric space is quite simple. Consider M a pointed metric space (i.e. with a distinguished point denoted 0) and $\text{Lip}(M)$ the space of all Lipschitz functions from M to $\mathbb{R}$ satisfying $f(0)=0$. It is well known that $\text{Lip}(M)$ equipped with the Lipschitz norm (the better Lipschitz constant of the function) is a Banach space. Now for every element x in M , we define the evaluation functional from $\text{Lip}(M)$ to $\mathbb{R}$ which associates any function f to its evaluation in x : $f(x)$. Of course those evaluation functionals are elements of the dual $\text{Lip}(M)^*$. Now the Lipschitz free space over M , denoted $F(M)$, is the closed linear span of those functionals. Of course this is a Banach space and actually $F(M)$ is a predual of $\text{Lip}(M)$, that is $F(M)^* = \text{Lip}(M)$.

A basic but important fact about free spaces is that they transform Lipschitz problems into linear ones through a commutative diagram. Thus it should be clear why it is interesting to explore their linear structure in order to face problems such as the one stated above.

Despite they are easy to define, their structure is quite difficult to analyze. For instance we still do not know whether $F(R^2)$ is isomorphic to $F(R^3)$. That is why a program launched by Godefroy and Kalton consist in the study of Lipschitz-free spaces over compact metric spaces or finite dimensional Banach spaces. The idea is to determine conditions ensuring that the free space has properties such as approximation properties (or even better the existence of a basis), Schur property, weakly sequentially completeness.

The aim of this presentation is to give an introduction to the study of Lipschitz-free spaces and then focus on conditions ensuring that those spaces have the Schur property.

Miércoles 26 de Octubre a las 16:30 hrs, Sala de Seminarios John Von Neumann CMM, Séptimo Piso d e Beauchef 851, Torre Norte.

