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**Seminar EDPs**

**Speaker:** Maria Westdickenberg (RWTH Aachen University, Alemania).

**Title:** Energetic methods for capturing sharp convergence- and metastable-relaxation-rates of gradient flows.

**Abstract:**

Together with Felix Otto, Richard Schubert, and other collaborators, we have developed energy-based methods to capture rates of convergence to equilibrium and used them to establish optimal, algebraic convergence for the Mullins-Sekerka problem in the plane and the Cahn-Hilliard equation on the line. The first method is based on the observation of Brezis that the gradient flow with respect to a convex (but not strictly convex) energy satisfies

$$E \leq (H\,D)^{\frac{1}{2}}, \quad \dot{E} = -D, \quad \dot{H} \leq 0, \quad \dot{D} \leq 0, \tag{1}$$

where $E$, $H$, and $D$ represent the energy, squared-distance to equilibrium, and dissipation, respectively. From this information it is not hard to deduce

$$E \leq \frac{H_0}{t}. \tag{2}$$

Our first result shows that it is possible to adapt this idea to the (mildly) nonconvex setting.

A second method that we have developed has at its heart a Nash-type inequality of the form

$$E \leq D^{\frac{1}{3}}(V+1)^{\frac{4}{3}}, \tag{3}$$

where $V$ represents the $L^1$-distance from equilibrium. Critical in order to make use of this information is a duality argument that establishes *boundedness* of the $L^1$-distance for all positive time. For Cahn-Hilliard on the line and "bump-like" initial data, this method captures *metastable* relaxation.

Work in progress with Richard Schubert (postdoc, RWTH Aachen University) combines elements from each framework.

[1] O. Chugreeva, F. Otto, and M. G. Westdickenberg, Relaxation to a planar interface in the Mullins-Sekerka problem, *Interfaces and Free Bound.* **21** (2019), pp. 21–40.

[2] F. Otto and M. G. Westdickenberg, Relaxation to equilibrium in the one-dimensional Cahn-Hilliard equation, *SIAM J. Math. Anal.* **46** (2014), pp. 720–756.

[3] S. Scholtes, F. Otto, and M. G. Westdickenberg, Optimal $L^1$-type relaxation rates for the Cahn–Hilliard equation on the line, *SIAM J. Math. Anal.* **51** (2019), pp. 4645–4682.

[4] S. Biesenbach, R. Schubert, and M. G. Westdickenberg, Optimal algebraic relaxation of "bump-like" solutions of the one-dimensional Cahn–Hilliard equation, submitted.

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Zoom Link

https://uchile.zoom.us/j/88968405982?pwd=Vy9icVA2NTROMTFoSzd6ZXlXaFFXdz09
Meeting ID: 889 6840 5982
Passcode: 309562
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**Thursday (6/24) at 4:15 pm (Chilean time).**