Journal of Convex Analysis
Volume 17 (2010), No. 3&4, 681–699

H. Attouch: To whom I am greatly indebted...

First of all, I would like to express my gratitude to the organizers of the meeting "Variational Analysis and Optimization," F. Alvarez, J. Bolte, A. Cabot, T. Champion, and M.-O. Czarnecki. Besides its high scientific interest, this meeting which was held in Montpellier in September 2009, offered me an exceptional opportunity to meet many friends, some of them I had not seen for a while. This meeting owes much to the support of the ANR and CNRS (that hosted us), to the laboratory I3M of Montpellier and his director B. Mohammadi and to the precious help of P. Redont, and B. Lacan. I would like to thank L. Thibault for publishing its proceedings in this special issue of Journal of Convex Analysis, a journal whose large international audience owes much to his management.

As it is becoming traditional for such (sixty) birthday events, I am going to evoke some stages of my forty years mathematical trajectory, and some of my favorite topics.

1. Early Training (1970-1982).

Without any doubt, for me, the crucial starting point was in dynamics. I was a student of the École Normale Supérieure (ENS Cachan), most of our courses were given at Université Paris-Sud, Orsay. After the agrégation in 1970, which was an obligatory step, I had to decide which direction to take. I was much attracted by analysis, certainly influenced by the course on integration theory by J. Deny, a prominent figure in potential theory, and by the clarity and elegance of the courses of F. Hirsch at ENS Cachan. At that time, analysis was mainly linear analysis, nonlinear analysis was only emerging as a new field.

J. Deny encouraged me to attend H. Brézis lectures, a "young promising mathematician" as he said, who was just starting teaching in Paris VI. As one can easily imagine, I got fascinated by his course, where functional analysis was married with such mysterious objects as multivalued monotone operators and lower semicontinuous convex functions, and which allowed to solve large classes of nonlinear PDE's. He introduced us to the very first results concerning semigroups of contractions in Hilbert spaces and the nonlinear version of Hille-Yosida theory. At the end of his course, H. Brézis proposed several subjects of research, all equally interesting. Indeed, A. Damlamian and I were interested in the same subject. The idea was, within a single differential inclusion, to combine monotonicity methods with Ky Fan methods, with an application to economics as proposed by C. Henry from the Laboratoire d'Economie de l'Ecole Polytechnique. Thus, we started working together on this subject under the supervision of H. Brézis. This was the beginning of a fruitful collaboration with a first joint publication in 1972 in the Israel J. Math. Meanwhile, in 1971, I was recruited, as what is now called Maître de Conférences, by Université Paris Sud, Orsay.

ISSN 0944-6532 / $ 2.50

Paris is an exceptional place for mathematics. I was rushing from one place to the other, from Orsay to Paris VI (Jussieu) and the Collège de France, to Dauphine, and then back to Orsay, and Ecole Polytechnique (by then settled in Palaiseau).

In Paris VI and Collège de France, the seminar animated by J.-L. Lions and H. Brézis has been playing an important role in the schooling of a whole generation of mathematicians. Attending the lectures given by some of the best mathematicians was a great motivation, but also a source of pressure, with as a challenge, trying to work at such a high level. The course of J.-L. Lions at Collège de France had a great impact on the development of applied mathematics in France, and made us familiar with the idea that mathematical developments should involve both theoretical and applicative aspects, and lead to numerical treatments. That's where we first learned about optimal control of distributed systems, singular perturbation theory, and homogenization theory of composite materials. From that period, I have kept friendly relationship with J.-B. Baillon, J.-M. Coron, T. Gallouet, and A. Haraux also students of Brézis, whose trajectories have been crossing mine several times.

As an original aspect of my trajectory in this mostly PDE world, I was much attracted by Paris Dauphine and the Cérémade seminary which was animated by J.-P. Aubin, A. Bensoussan, P. Bernhard, I. Ekeland, J.-M. Lasry, ... Indeed, as it was usual at that time in Paris, the "thèse d'état" involved a second subject, selected from a different domain than the main thesis, to be explored in a relatively short time delay. Mine came from cooperative games under the supervision of J.-P. Aubin. This has been for me an exceptional chance to discover the Cérémade, a place where some of the modern tools of mathematics of decision theory were being built. J.-P. Aubin is a creative mathematician, an explorer of new applicable domains of nonlinear analysis such as biology, neural sciences, robotics, and, he did so, with a communicative enthusiasm. I guess I have inherited from him the conviction of the importance of set valued analysis and differential inclusions. He had remarkable PhD students and with some of them I have kept a close relationship, all along, B. Cornet, P. Saint-Pierre, M. Quincampoix, P. Cardaliaguet, and H. Frankowska (a former student of C. Olech). The colloquium dedicated to his 65th birthday in Roscoff, June 2004, has been an exceptional occasion to meet all of them in a friendly challenging atmosphere, just like it was in Dauphine 30 years earlier. In Dauphine, I had the chance to share some rare moments, like the one where I. Ekeland explained to me his recent discovery of what was going to be his celebrated ϵ-variational principle. All along the years, his monograph with J.-P. Aubin on applied nonlinear analysis, Wiley 1984, has been one of my favorite reference books. We had the chance to learn from I. Ekeland, the optimization and equilibrium concepts (Pareto, Nash) underlying mathematical modeling in economics. Quite amazed, I realized that mathematical modeling in economics, games, decision sciences often uses similar objects as mechanics and physics like variational methods, convexity, set-valued operators, differential inclusions, dissipative dynamical systems... Thus, I convinced myself that optimization (being understood in a large sense, i.e., involving both finite and infinite dimensional problems) and dynamical systems are central topics in mathematics and its applications.

Indeed, Orsay was a special place for me. It was the place where I taught, which allowed me to get fruitful exchanges with J.-M. Bismut, J.-M. Bony, M. Demazure, J. Deny, A. Douady, G. Ioss... I participated to the seminar of R. Temam which was

focused on Navier-Stokes equations and PDE's from plasma physics. I had also the chance to attend the courses of L. Tartar, especially on nonlinear hyperbolic systems. Discussions with L. Tartar were always very fruitful, and we were all impressed by the depth of his comprehension of Physics and his ability to translate it mathematically.

Most important, we had a working seminar in Orsay which was organized by J. Deny, F. Hirsch and Ph. Bénilan. In a friendly atmosphere, without time constraint (it took place every Saturday morning!), it was a place of intense exchange where we have been learning together nonlinear semigroups theory and its connections with potential theory. We had the exceptional chance to attend lectures given by G. Choquet and L. Schwartz who, as friends of J. Deny, accepted to come to our informal seminar. A. Damlamian, C. Picard, M. Pierre and I had regular exchanges with Ph. Bénilan. We owe him much for his advices, encouragements. I remember particularly well him saying, "don't hesitate to generalize, and go to an abstract formulation, so as to find the underlying fundamental concepts." At that time, this sentence seemed to me contradictory with the overall message from J.-L. Lions. It took me some time before discovering that they were not really contradictory. Even after he left Orsay to accept a Professor position in Besançon, we have kept a close relationship, and collaborated on a couple of papers. In 1977, he has been organizing a memorable congress on Nonlinear Analysis in Besançon, which has been for me the occasion to meet J. Ball, G. Duvaut, J.-P. Gossez, U. Mosco, L. Nirenberg, O.A. Oleinik, ... We have been sharing mutual friends with the Spanish analysis community, including J.I. Diaz, L. Vasquez, which led us later to meet several times in Madrid. I have been returning later to Besançon for the PhD thesis of P. Wittbold, one of his many remarkable students, as they were working on variational sum of accretive operators. P. Wittbold is now University Professor in Berlin.

In June 1976, I defended my "thèse d'état" in Paris VI under the supervision of H. Brézis. Half of the thesis was composed of joint papers with A. Damlamian, one of them was a seminal work concerning evolution equations governed by time-dependent subdifferential operators. The other half was composed of results concerning variational convergences, a new emerging promising field. One of the referee of my thesis was Ch. Castaing, who invited me to come to Montpellier to discuss some aspects of my thesis. Indeed, he asked me to give him another proof of a result which was not clear to him. Fortunately, I succeeded at this test, during a dinner at a Chinese restaurant in Montpellier. A few days after defending my thesis I got married with Annie, who being, a geographer, encouraged me to develop an active traveling research program.

In 1979, I got a six months post-doctoral grant CNRS-NSF which permitted me to visit several universities in the United States and got fruitful exchanges: Courant Institute (L. Nirenberg), Madison-Wisconsin (M. Crandall), Chicago (F. Browder), Lexington-Kentucky (R. Wets), Washington-Seattle (R.T. Rockafellar). This was an important experience for me which made me discover and experience the way research and teaching was organized in the United States.

From that date, I have been especially developing an active cooperative research with R. Wets touching several interconnected domains like epigraphical analysis, quantitative stability analysis of variational systems, variational convergence of bivariate functions, random multivalued processes and the epigraphical law of large numbers. I

have been learning a lot from R. Wets, our joint production has much benefited from his deep knowledge of stochastic optimization and of its applications to the economical world. From the beginning, R.T. Rockafellar has been very attentive to my research progress, and we have been sharing the same strong interest in convex analysis and optimization. During my stay in Seattle, I have also learned a lot from him about the beauty of the mountains and particularly of Mont Rainier. Madison-Wisconsin was also a very interesting place. The MRC (Madison Research Center) was actively visited by a number of French mathematicians. J. Nohel, who was the chairman, had a communicative enthusiasm for integro-differential and Volterra equations, that's the place where I got familiar with the subject, and produced a couple of papers on the subject. It was also the place where I met M. Crandall who was actively working with Ph. Bénilan. I have the manuscript of their book, a priceless document, which as far as I know has never been published.

The Italian school had also a great influence on me. I first visited Rome in 1978 and 1979 where I met U. Mosco, who impressed me by his large culture and his deep comprehension of mathematical analysis. At that time, I was discovering the rich connections between Mosco convergence of sequences of convex functions, the graph convergence of the associated sequences of subdifferential operators, and the notion of measurable integrand. These topics were directly linked to subtle questions concerning variational inequalities with thin or variable obstacles, capacity and potential theory, free boundary problems. I was precisely working on these questions with C. Picard, and we got fruitful exchanges with young Italian colleagues working on similar subjects L. Boccardo, M. Biroli, I. Cappuzzo-Dolcetta, ... In France, A. Damlamian, M. Pierre, C. Picard and I benefited too from discussions with J.-L. Joly who was in Bordeaux and had been working with U. Mosco on these subjects. His thesis was concerning continuity properties of the Legendre-Fenchel transform in general topological vector spaces. Unfortunately, at that time, U. Mosco and J.-L Joly were turning their attention to different domains. I can imagine that, at this moment, they failed to have enough motivating applications in order to develop the subject further. I also met G. Stampacchia, who, despite his celebrity, impressed me by his simplicity. He was close to H. Brézis, and I had the chance to talk to him. He was kind enough to give me the impression he was interested with my thesis results about variational inequalities. His sudden death a couple of years later, as he was visiting Paris, was a great shock and a grievous loss for the mathematical community.

A decisive encounter took place in Pisa, where in 1981 and again in 1982, I met E. De Giorgi who was founding the general topological theory of Γ-convergence, and thus considerably enlarged the framework of variational convergences. An exceptional coincidence was the simultaneous emergence of the homogenization theory which aims at studying the laws of physics in composite materials from a macroscopic point view, by defining equivalent homogeneous materials. The point was that Γ-convergence helped understanding the mysterious convergences and formula underlying the homogenization processes. There was much excitement about the new world we were discovering, with such exotic objects as the fakir's carpet, the cloud of ice and the strange terms appearing in the limit equations. This terminology has been introduced by D. Cioranescu and F. Murat in an important seminal paper related to these questions. From that date, I more or less integrated with the group of De Giorgi in Pisa and built friendly

mathematical relationship with G. Buttazzo, L. Carbone, G. Dal Maso, De Arcangelis, L. Modica, C. Sbordone, and a little bit later with L. Ambrosio. I got familiar with the Scuola Normale Superiore di Pisa and the Universita di Pisa. During my many stays I also got fruitful exchanges with S. Spagnolo, A. Marino, M. Degiovanni. I had interesting exchanges with P. Marcellini around Vectorial Variational Analysis, which made me also meet B. Dacorogna (in Lausanne), a remarkable mathematician and person. From that period, I have kept close relationship with G. Buttazzo with whom, (and G. Michaille), I wrote later (2006) a monograph published by SIAM on "Variational Analysis in Sobolev and BV spaces."

2. Next phase and new responsibilities (1983-2004).

I had been staying around ten years in Orsay as Maître de Conférences. This looks like a rather long period, but at that time, careers were completely frozen and very few university positions were available (a cyclic phenomenon in France). I experienced the fact that academic careers, probably because of their human components, sometimes obey unpredictable laws. Quite often, in the recruiting process, small effects can dramatically affect the final choice. I was about to join Lyon and J.-B. Baillon, but curiously the fact that my coming was favorably considered both by the fundamental and applied analysis groups became a handicap. Indeed, at the last moment, some applied mathematicians in Lyon feared that there was the risk that I would indeed do mostly theoretical things!

2.1 Perpignan. As a result, in 1983, I ultimately went to Perpignan as Professor of Mathematics, and joined A. Fougères who was interested in developing new ideas around integral functionals and variational analysis. Indeed, A. Fougères proposed setting up a new laboratory centered around these themes and offered that I take its direction. This turned out to be an exciting, but a demanding experience. The result was the creation of the AVAMAC laboratory, an abbreviation for "Analyse Variationnelle and Applications to Mechanics, Automatics and Control". One of our guideline was to develop collaborative research with colleagues in Perpignan working on automatic and control of solar energy systems, Amouroux, El Jai, ... A. Fougères had many other ideas and projects, and very soon we created an active group of young mathematicians, including besides A. Fougères and myself, D. Azé, G. Bouchitté, J.-C. Peralba, A. Truffert. I had many PhD students, S. Abdulfattah and M. Soueycatt from Syria, E.H. Chabi and M. Hnid from Morocco, J.L. Ndoutoume from Gabon, with whom I have kept in contact for a long time. We benefited from a strong support of the university administration who was interested in developing an active research group in mathematics with interdisciplinary aspects.

D. Azé was a former student of I. Ekeland. His PhD thesis in Dauphine concerned variational methods for Hamiltonian systems. I. Ekeland encouraged him to join our group in Perpignan, where after his coming, he turned his attention to variational convergences. He prepared his "thèse d'état"under my supervision on the quantitative stability analysis of variational systems. During the five years (1983-88) I have been staying in Perpignan, we had a friendly fruitful scientific collaboration, and he helped me efficiently in the organization of the laboratory.

By chance, G. Bouchitté, after he got his engineering degree in Paris, and after a one year round the world trip, was back to Perpignan where he had family. He had started working with A. Fougères on a variational approach to the Plateau problem (non parametric minimal surface problem) based on a relaxation-projection method using the bidual. He was stuck in this difficult problem, and A. Fougères asked me to propose him another, hopefully more accessible problem. Based on his experience of variational analysis in nonreflexive Banach spaces, I proposed him to work on homogenization in plasticity, in BV spaces. He defended his "thèse d'état" in 1987, and that was the beginning of a remarkable career. I now realize that my major contribution was to encourage him and to help him discovering his exceptional capacities.

During this period I have been actively promoting the new emerging notions of convergence for variational problems: In 1984, a few time after my arrival in Perpignan, I finished writing the book "Variational convergence for functions and operators" which was the first monograph on this subject. It has been published by Pitman, and it offered a synthetic view on Γ-convergence, Mosco convergence, graph convergence of monotone operators, and their applications.

In 1987, I have been giving a lecture at Séminaire Bourbaki in Paris about Homogenization. I must confess that I have rarely been so stressed, there was palpable excitment in the audience, the next speaker being Jean-Pierre Serre!

In 1986, I wrote with H. Brézis a paper, introducing a new, so-called constraint qualification, which allows to compute the conjugate of the sum of two convex functions in Banach spaces. This condition is now known as the Attouch-Brézis qualification condition. It has been first motivated by a question of H. Brézis, who, as he was writing his remarkable book on Functional Analysis (Masson, 1983), asked whether it was possible to extend to a nonlinear convex setting the classical linear duality results for the orthogonal of the intersection of two closed subspaces. Afterwards, this result has been the occasion of fruiful exchanges with S. Simons, when I visited him in Santa Barbara, California. His recent monograph (Lecture Notes in Math., 2008) gives an excellent account of this subject and of some related questions for monotone operators.

During this period, an important event was the organization, in June 1987, in the "Palais des Congrès de Perpignan", of the "Congrès Franco-Québécois d'Analyse Nonlinéaire Appliquée". This meeting was organized by J.-P. Aubin, F. Clarke, I. Ekeland and myself. It benefited from the support of the Office de Coopération Franco-Québécois and of his secretary Ph. Bergeron. Besides the participation of a strong Canadian group including F. Clarke, J. Gauvin, M. Fortin, G. Fournier, we succeeded gathering many of the best mathematicians working on that domain. Maybe it was one of the last event of that type, because very soon applied Nonlinear Analysis splitted into many specialized domains like PDE's, control, computational methods, optimization, operational research, games, ... I remember that this event took place just at the same time as the "fête de la musique", with a friendly atmosphere. A. Ioffe, who was one of the lecturer, was not able to come because of limitation of freedom in Soviet Union, and J.-P. Aubin, as a symbol, read his lecture in his place. The proceedings of the congress appeared in one of the first volumes of Annales de l'IHP, a journal that I. Ekeland was launching.

Being in Perpignan, an important aspect was the close relationship with the Laboratoire d'Analyse Convexe de Montpellier. We had regular exchanges with Ch. Castaing and M. Valadier around integral functionals, measure theory, Orlicz spaces. With the geometers of Montpellier we have been participating to the creation of a doctoral program including Montpellier, Perpignan, and a little later Avignon.

Even after I left, I have kept close relationship with Perpignan, especially with D. Aussel, and J.-N. Corvellec, who have been maintaining in Perpignan an excellent research activity around nonsmooth analysis and critical point theory. I have also kept friendly relationship with H. Bonnel, who is now professor at the Université de Nouvelle-Calédonie, Nouméa, and J.-A. Marti, professor at the Université des Antilles-Guyanes. As Salvador Dali said, the "gare de Perpignan" is the center of the world!

2.2 Montpellier. In 1988, after five years in Perpignan, Ch. Castaing and M. Valadier proposed a transfer to Montpellier which I accepted.

At that date, J.-J. Moreau, who was at the origin of the creation of the Laboratoire d' Analyse Convexe, with Ch. Castaing and M. Valadier, decided that, indeed, he was mostly a Mechanics person. Thus, he joined the "Laboratoire de Mécanique et Génie Civil" of Montpellier, in short LMGC, where he turned his activity towards numerical simulation of granular materials. Despite his close and friendly presence, analysts in Montpellier felt a little bit like orphans. On a counterpart, they got from him an exceptional heritage. Besides his fundamental contribution to convex analysis, many of the concepts and tools he has introduced in nonsmooth mechanics and unilateral analysis, like "problème de rafle, dynamics with acceleration measure" have become classical.

Soon after my arrival, A. Brillard, defended in 1989 at Montpellier his "thèse d'état". Under my supervision, he had been working on the homogenization of some equations of continuum mechanics in porous media. As a major difficulty in the case of systems, and by contrast with scalar equations, maximum principle and classical truncation methods do not apply anymore. A. Brillard has been pursuing his career at the Université de Mulhouse, where, as president, he has been assuming important responsibilities.

In 1992, Ch. Castaing retired and he asked me to take the direction of the Laboratoire d'Analyse Convexe, with as new challenge, enlarging its scope. I assumed this function until 2004, at which date all the mathematical laboratories of Montpellier joined themselves into a single institute, I3M, namely Institut de Mathématiques et Modélisation de Montpellier. During these 13 years, from 1992 until 2004, a large part of my activity has been devoted to develop the Laboratoire d'Analyse Convexe. I have been very attached to the human aspects of this function. In particular, I have always tried to associate each member of the laboratory to the decisional process, so as to create a team spirit.

B. Lemaire, who got his thesis under the supervision of J.-L. Lions, complemented the professoral group. Very soon we had fruitful exchanges about continuous optimization, and we have been sharing the same intuition concerning the importance of proximal algorithms.

In 1994, L. Thibault, who was a former student of Ch. Castaing, returned to Montpellier, in a Professorship position (transfer from Pau). He soon became the main animator of the historical orientation of the laboratory. We also had a solid group of Maître de Conférences, including M.-F. Nougues Sainte-Beuve and J. Saint-Pierre, both graduates from ENS. They are both talentuous analysts who have limited their career ambition, mainly for family reasons.

In 1995, R.T. Rockafellar was declared an Honoris Causa doctorate by the University of Montpellier. All of our group actively participated to this important event, that underscored our appreciation of the fundamental contributions of R.T. Rockafellar to convex analysis and optimization.

We had the chance to obtain rapidly a Maître de Conférences position, which allowed us to recruit G. Michaille, a former student of M. Chipot (who himself is a former student of H. Brézis!). This was an excellent operation, G. Michaille combines scientific with rare human qualities. He soon played a major role in the animation of our Variational Analysis group, and developed a strong connection with our colleagues from mechanics, especially with Ch. Licht, and further with F. Krasucki. He got promoted to Professor at the Université de Nîmes, an institution which had just been created, while maintaining his research activity in our team.

I have always been very attentive to maintain a strong connection with our colleagues from Mechanics, a policy that I have been sharing with O. Maisonneuve, who has been responsible for mechanics in Montpellier for many years. From this, has been emerging a rich active net of scientific relations with P. Alart, G. Geymonat, F. Krasucki, Ch. Licht, O. Maisonneuve, J.-J. Moreau in Montpellier, P. Ballard, A. Léger, P. Suquet from the LMA in Marseille, M. Frémond from the Laboratoire Central des Ponts et Chaussées in Paris, A. Cimetière in Poitiers, ...

Among the most important decisions was the creation in 1994 of the Journal of Convex Analysis. Indeed, the activities of the laboratory already benefited of a large international audience via the "Travaux du Séminaire d'Analyse Convexe". M. Valadier was very attentive to the quality of its publications. We decided to formalize the situation in accordance with the publication standards of an international journal, and so was born the Journal of Convex Analysis. Ch. Castaing had personal contacts with a German editor, Heldermann, who, as a distinctive feature, is a mathematician who is specialized in mathematical editing, and thus became our editor. The managing editorial board was composed of Ch. Castaing, G. Buttazzo and R. Wets. When Ch. Castaing stopped his activity at the direction of the journal, L. Thibault accepted to take its direction, a rewarding but very demanding activity, so keeping in Montpellier the heart of the journal. One should mention that, from the beginning, our secretary, B. Lacan has been playing an important role in the production of the journal. Only the editing part of the Journal is not done in Montpellier but namely in Berlin. It is a great satisfaction to say that the journal enjoys an excellent international reputation, as attested by its ranking, and its enlarging range of activity.

It has been a sensitive decision to enlarge even more the topics of the laboratory. From the beginning, I have been convinced of the importance of associating theoretical and applied activities within the mathematical analysis laboratory. In particular, this permits to offer to the students a large spectrum of choice for further research, and professional activity. Indeed, Montpellier had missed the numerical analysis (theoret-

ical) phase, and thus we decided to pass directly to the next phase which is scientific computing, with the stress on the relationship with industry and economics world.

That's why, in 1998, we recruited B. Mohammadi, who has been in charge developing these applied aspects. B. Mohammadi was a former PhD student of Pironneau. He came from INRIA, and was strongly supported by J.-L. Lions who recognized him as one of the best specialist in automatic differentiation for large-scale optimization, numerical fluid mechanics, and with striking applications to optimal shape design in aeronautics. This turned out to be a successful operation, B. Mohammadi revealed himself as a friendly person, with easy direct access, and we soon developed collaborative projects. In order to consolidate this topic, immediately afterward we have been recruiting as Maître de Conférences, B. Koobus, a former PhD student of Dervieux (INRIA, Sophia-Antipolis). Three years later (2001) we have been recruiting as professor F. Nicoud, who came from the CERFACS Toulouse, and is an excellent specialist of Thermoacoustic instabilities, Combustion noise, and Cardiovascular biomechanics. This created a great dynamic, as attested by the coming in our laboratory of M. Cuer, who originally was in the Laboratoire de Physique Théorique de Montpellier, where he was working with P. Sabatier, the world-known specialist of inverse problems. As another successful but quite difficult operation, where we have been experimenting the inertia of the French administration, we have been obtaining the transfer to our laboratory of P. Redont, a former research engineer at CINES, more attracted by mathematics and numerical analysis than by maintenance of big computers for massive computation.

We also have been creating an interesting connection with medicine, around medical imaging and tomography. J.-P. Crouzet and P. Maréchal (who is now professor at Toulouse) and Professor D. Mariano-Goulard from the CHU of Montpellier have been the main animators of this fruitful and still active collaboration.

Montpellier is an important resarch and development center for Agronomy, Environment and Biology. Around these subjects and within the MERE project, we have been developing an active collaboration with A. Rapaport and F. Mazenc from the INRIA Montpellier, "UMR Analyse des Systèmes et Biométrie", and further with Cl. Lobry. One of our main concern is the control of bioreactors.

This strong mutation led us to change the name of the laboratory, and I proposed the name ACSIOM, an abbreviation for "Analyse, Calcul Scientifique Industriel, et Optimisation de Montpellier," which was a synthesis of the now theoretical and applied aspects of the laboratory. This evolution was encouraged by the CNRS, and by the successive responsables of the mathematical department of Montpellier, D. Guin, G. Laffaille, J. Lafontaine, Cl. Cibils. The most recent step of this evolution has been the development of the PDE thematic with first the recruiting of J. Droniou as Maître de Conférences, followed by his promotion to Professor.

Besides the opening of these new themes, we have been attentive to maintain the historical orientation around convex and nonsmooth analysis. The recruiting of M.-O. Czarnecki, who was a former student of B. Cornet and then his promotion to Professor, together with the recent transfer from Limoges of A. Cabot (one of my best former PhD students), and the recruiting of T. Bayen (a former PhD student of T. Lachand-Robert, specialist in shape optimization and optimal control), should maintain the high quality of this orientation.

It would be too long to cite all the recruiting operations, but let us say that it is a great satisfaction to see that we have mostly succeeded in this evolution, while taking account of the human and family aspects. Indeed, we have two couples of mathematicians in our laboratory, namely C. Lacour and B. Koobus, and O. Iosifescu and P. Azerad, who all have excellent mathematical activity.

International relations, especially with Chile, Italy, and Morocco have been very active during this period, and formalized by several international conventions of cooperation. Collaboration with University of Pisa and G. Buttazzo has been very fruitful. As a result G. Buttazzo, G. Michaille and I wrote a monograph of more than 600 pages on "Variational Analysis in Sobolev and BV spaces", which has been published in 2006 by SIAM. The first part of the book is the basis of the graduate course on variational analysis and PDE's in Montpellier and Pisa, the second one presents some recent advances in the domain.

I have always been attentive maintaining close relationship with the other mathematical teams in Montpellier, geometers, algebraists and statisticians, and thus acting for their reunification into a single institute, I3M, in 2004. By many aspects this has revealed to be a positive decision. The new institution has been strongly supported by the CNRS, as attested by the recent coming of R. Carles, from CNRS, a PDE specialist. But that is another story, where B. Mohammadi the actual director of I3M did an excellent job.

As a guideline, all along this evolution process, I have been looking to maintain equilibrium, or to reach for new ones, thus experimenting pratically with Nash equilibrium, a topic whose importance I was just discovering.

2.3 Developing Optimization in France. Development of Optimization (and related topics) in France has been an exciting adventure. I participated in one of the first meeting in St. Pierre de Chartreuse (close to Grenoble) organized by P.-J. Laurent, followed by the meeting in Murat Le Caire (organized by J.-P. Aubin and A. Auslender). But soon after, some of the main animators of this orientation have been following their own trajectories. Indeed, there was a real need for coordination and representation of this area in the national environment. Soon, I have been convinced of the importance of this task, and I have been defending the optimization community, which was organizing as a new group in SMAI (Applied Math. Soc.), namely MODE (Math. of Optimization, Decision, and Economics). At the CNU (the National Committee for Universities which takes care of careers and promotions), I have been sharing with B. Cornet and M. Bergounioux the difficult task of defending this relatively small community (by comparison with the PDE and Statistics groups).

From the beginning, J.-B. Hiriart-Urruty has been playing an important role in this process, both by the quality of his mathematical production, and by his active role in the decision centers. The nomination at the direction of SMAI in 2001 of M. Théra, a mutual friend, who is issued from the optimization and variational analysis community, attested the growing importance and quality of the group MODE.

M. Théra is a person whose qualities I appreciate very much. Indeed, I had the opportunity to better know him during one of our stay at the University of Davis, California, where I visited R. Wets. Since then, we have established a friendly relationship,

and we have been writing several papers together. One of them concerns the notion of variational sum for maximal monotone operators, a paper we have been writing with J.-B. Baillon. Another one concerns a general concept of duality for nonlinear problems, which encompasses all the classical duality relations like Fenchel, Toland and Clarke-Ekeland. Indeed, we have been completing this last paper while we both visited F. Clarke at the University of Montréal. I have always kept a strong connection with Limoges. I have known the initial period where a remarkable group of young mathematicians was animating research in Nonlinear Variational Analysis in Limoges, namely M. Théra, M. Volle, J. Blot, Ch. Malivert, J. Guillerme, and a little later J. Benoist. I have high regard for all of them. I have often returned to Limoges, notably as president of the jury of the HDR thesis of D. Goeleven (1997), and further of S. Adly (2005). S. Adly is a nice mathematician and person with whom (and A. Cabot) I have been writing a paper on the finite time stabilization of nonlinear oscillators with dry friction, a paper which appeared in the volume celebrating the 80th birthday of J.J. Moreau. The last time I went to Limoges was for the sixty birthday of M. Théra, which has been an occasion for meeting a number of mutual friends, G. Beer, R. Lucchetti, J. Revalski, S. Simons, ...

When M. Volle has been nominated Professor at Avignon, we have developed an active research program relying Avignon and Montpellier based on a joined doctoral program. We have been writing together a paper devoted to cutting and scanning methods in set-valued analysis. After A. Seeger joined him in Avignon in a Professorship position, they have been able to create an active research center in optimization and nonlinear analysis. I have been very happy when M. Volle has been elected as president of the University of Avignon, a position where he has been able to exploit his exceptional human qualities and his high scientific vision.

I had already several opportunities to cite F. Clarke, with whom I have kept a close friendly relationship, all along. I remember well this exceptional moment (in 1973) when J.-P. Aubin, who was just back from North America, announced in Collège de France, that a young canadian mathematician, named F. Clarke, had discovered a notion of derivative for nonconvex Lipschitz functions which enjoyed as many desirable properties as in the convex case. Exceptionally, the week after, the seminar schedule was modified so as to allow J.-P. Aubin to explain us what was going to be the Clarke's derivative. In 1994, F. Clarke left Montréal and came to Lyon in a Professorship position. His nomination in 2000 at the "Institut Universitaire de France" has been supported by the optimization community (J.B Hiriart-Urruty and I had the charge and honor to propose his nomination). Since, he has developed a remarkable research group in control theory. F. Clarke is an exceptional lecturer, his courses at ENS Lyon had a great impact on a whole generation of young mathematicians, P. Bousquet, T. Champion, C. Imbert, O. Ley, L. Rifford... Afterwards, all of them have been bringing rich life to the optimization and control communities in France. The meeting which has been organized in Rome in May 2009, for his 60th birthday has been an exceptional mathematical event, showing the large and deep contributions of F. Clarke to control theory, nonsmooth analysis, calculus of variations, and optimization.

The activity of the optimization community has been organizing around some periodic events, like the congress of the group MODE, the French-German conference on Optimization, the French-Chilean conference on optimization, ... Each of these

meetings has been the occasion to meet many colleagues and friends. Besides the ones I already or further mention, and as being close to my generation, I think to M. Bergounioux, F. Bonnans, J.-M. Bonnisseau, P.L. Combettes, J.-P. Crouzeix, M. Lassonde, D. Noll, J.-P. Penot, S. Sorin... With each of them I have been sharing memorable moments:

J.-P. Penot has been one of the first organizer of a congress in Nonlinear Analysis. It was in Pau in 1972. I remember that, during the excursion by car to Gourette-Artouste in the Pyrénées, H. Brézis was inspecting all his students, including me, asking them (kindly) about their recent progresses. J.-P. Penot is a talentuous analyst who has been coming from geometry. He possesses a large culture and an acute taste for nice mathematics. We had the pleasure to write a joint paper with one of my former PhD student, H. Riahi on the continuation method for variational problems (since then, H. Riahi has returned to University of Marrakesh, Morocco).

I have been knowing well J.-P. Crouzeix and his brother Michel, both talentuous mathematicians. With J.-P. Crouzeix, I have been sharing several friendly mathematical and cultural discussions while both of us visited our favorite optimization centers, like Chile.

M. Lassonde (like J.-N. Corvellec and Ch. Horvath) is a former PhD student of A. Granas. I visited him several times in Pointe-à-Pitre, Université des Antilles-Guyanes, where with R. Janin, O. Nakoulima, and other talentuous mathematicians he has developed an excellent group in optimization and nonlinear analysis. I have also considered with great interest the development of a remarkable group in Pointe-à-Pitre, working around algebra of generalized functions, and organized around J.-A. Marti (whom I knew from Perpignan) and A. Delcroix.

The future is also nice for the new generation D. Aussel, T. Bayen, J. Bolte, A. Cabot, G. Carlier, T. Champion, J.-N. Corvellec, M.-O. Czarnecki, J. Malick, P. Maréchal, ... I have high regard for all of them. G. Carlier is being acting as a remarkable director of group MODE. The "Séminaire Parisien d'Optimisation" at IHP (animated by J.-B. Baillon, F. Bonnans, P.-L. Combettes, G. Carlier, and S. Sorin), and the recent creation of the GDR Optimization are important successes. One of my regret is not to be able to cite here T. Lachand-Robert, an exceptional mathematician and person who has left us much too young.

3. Some of my favorite topics.

Here comes a difficult exercise, which consists selecting some of my favorite topics, and some of my contributions to these subjects. Each mathematician has his own appreciation of what is nice mathematics. Because of my background, which is at the interface between theoretical and applied mathematics, I like mathematical concepts which are simple in their formulation, and which allow for a wide range of applications. Of course, looking for simplicity of the concepts and statements means extracting the basic underlying structures. Aiming at a large range of applications reflects the universality of the mathematical concepts, and allows for rich exchanges between different domains. I have experienced these facts while using very similar tools in mechanics, physics, economics and cognitive sciences. Finally, I have selected two topics among my favorite ones.

3.1. Variational convergences, Set convergences, and Approximation methods. I have always be fascinated by the joint development of Variational Analysis and Functional Analysis during the 20th century. It started with the initial intuition of D. Hilbert (1900, Collège de France conference), followed by the constructions of H. Lebesgue (integration) and L. Schwartz (derivation). These have been the basic blocks permitting the development of the theory of Sobolev spaces, and further of the variational approach to a large families of linear PDE's.

When passing from linear to nonlinear problems, I have been learning from H. Brézis the important role played by monotonicity and convexity structures (quite often, convexity is present but hidden as Y. Brenier has recently pointed out). H. Brézis made me discover subdifferentials of convex lower semicontinuous functions, which is an important class of maximal monotone operators. From the beginning, I have been very interested by this class of operators, which naturally comes into play when writing optimality conditions for convex variational problems, and thus plays a central role in optimization, mechanics, PDE's, ... But working with such class of operators means dealing with nonsmooth, possibly multivalued objects!

In my investigations, I have been guided by the analogy with distribution theory, which also involves singular objects, and where convolution allows to regularize them, thus reducing analysis to the classical one. First, I have tried to understand the approximation of subdifferentials of convex lower semicontinuous functions by smooth operators with as guideline the Yosida approximation. I knew the right concept of convergence for sequences of operators, which was the resolvent, or equivalently the graph convergence. That's the way I discovered the corresponding notion for sequences of convex functions. It was Mosco-convergence. I made further the link with the Γ-convergence of De Giorgi, thus extracting the basic topological concepts underlying variational convergences. This has been the object of the monograph I published in 1984 on variational convergences (Pitman editor).

With R. Wets we have been putting to the fore the central role played by the epigraph in the geometrical interpretation of these notions of convergence (for minimization problems), whence the terminology epi-convergence. As soon one is familiar with these relatively simple notions, one has in hand a powerful tool in order to understand in a unifying way the convergence properties of many approximation or perturbation schemes. For example, monotone convergence of a sequence of functions automatically implies its epi-convergence, and hence the convergence of solutions of the corresponding problems. This yields general convergence results for penalization schemes, barrier, and viscosity methods. Continuity properties of the Legendre-Fenchel transform for topologies associated to epi-convergence allows to handle simultaneously convergence properties of dual objects, multipliers, ... Even more, these notions are directly linked to the convergence of the associated semi-groups, and evolution problems.

During the period 1980-1993, my collaboration with R. Wets in this area has been extremely productive. We have been writing no less than 16 papers, which means a great complementarity, shared mutual enthusiasm and perspectives. A large part of this program has been achieved while I visited University of Davis, California, where R. Wets had his position. We also met at IIASA, an international institute, in Laxenburg, close to Vienna, Austria, where we have been spending some summers working

together. We have been developing new concepts, like variational convergence of bivariate functions for studying approximation and stability of saddle value or min/sup problems, namely epi/hypo convergence. Part of this program concerning links with duality and its continuity properties has been further developed with D. Azé. We have developed epigraphical analysis as a new branch of mathematical analysis which is well adapted to optimization, with its own topological, differential, and integration concepts. As an anecdote, we have been introducing new terminologies, some of them have been successful, like Moreau-Yosida approximation, epigraphical sum, epi/hypo convergence, ...

The basic monograph of R. Wets and R.T. Rockafellar devoted to "Variational Analysis", Springer 1998, reflects well the two complementarity approaches to nonsmooth variational analysis, let us say the intrinsic one relying on the concepts of generalized derivatives, cones, ... and the approximation, perturbation approach, which we have been developing with R. Wets. It is an opportunity to insist on the close relationship between the optimization and the nonsmooth analysis community, and the important contributions to these domains of J.M. Borwein, F. Clarke, J.-B. Hiriart-Urruty, A. Ioffe, B. Mordukhovich, J.P. Penot , R.T. Rockafellar, L. Thibault, for whom I have high regard.

Passing from the topological to the metric, quantitative aspects has been an essential step of our program. It was essential to quantify the proximity of two functions or operators for the numerical applications of these methods, and their practical use. The concepts we have introduced for the quantitative stability of variational systems rely on the localization of the Hausdorff metric, and the epigraphical and graphical geometrical view of functions and operators. They have become popular and often referred to as Attouch-Wets convergence, or Attouch-Wets topology. This last topological notion has been developed with R. Lucchetti during one of our joint visit in Davis.

One of the paper with R. Wets I like very much, concerns the epigraphical law of large numbers. It has been first proposed to one of the best journal of probabilities, unsuccessfully. We have been lazy and the paper remained as Publication of the Séminaire d'Analyse Convexe de Montpellier. I have been very pleased to discover recently that colleagues in statistics in Montpellier knew the paper and appreciate it! As a rigid rule, I have observed that it is difficult to publish in a domain which is different from the one you come from!

While visiting California, I met G. Beer who was working on set convergences and related topologies. G. Beer is a topologist, and combining our different perceptions and motivations turned out to be very productive. He visited Montpellier a couple of time. In 1993, we obtained a nice result concerning the convergence of subdifferentials of convex functions in general Banach spaces, which was published in Arch. Math. As a key ingredient, the paper used slice convergence, a new notion introduced by G. Beer. As another important topic, G. Beer has notably enlightened the topological notions underlying the "Attouch-Wets"convergence. With this respect one should also mention the contributions of J.-P. Penot, R. Lucchetti, Y. Sonntag and C. Zalinescu. The monograph published in 1993 by G. Beer (Kluwer) on "Topologies on closed and closed convex sets" gives an excellent account of this rich subject.

One of the great interest of variational convergences is to produce and create new mathematical objects, just like singular perturbation or relaxation methods do. That's the way, in a joint work with M. Théra and J.-B. Baillon (JCA, 1994) we have been able to define a generalized notion for the sum of maximal monotone operators, as a variational limit of a regularized expression. This approach offers new perspective on an important subject, related to the Trotter-Lie-Kato formula, and Schrödinger equations with singular potentials.

In 1994, the CIRM conference that has been organized by M. Théra and myself centered on these subjects reflecting well the rich diversity of the subject. Among the participants, who were coming from different domains, there was Z. Artstein, E. Balder, R. Deville, Ch. Hess, R. Lucchetti, R.A. Poliquin, S. Robinson, C. Zalinescu, T. Zolezzi, ... Its proceedings have been published in volume 2 of Set-valued Analysis, a journal which was just launched by B. Ricceri.

3.2. Dynamical systems, algorithms and optimization (1996-2010). While parallely progressing in optimization and variational analysis of PDE's, I gradually discovered the importance of gradient systems, and their great flexibility. Using subdifferentials and corresponding differential inclusions permits to enlarge their range of applications to several domains like PDE's and unilateral mechanics. Viewing numerical algorithms as discrete dynamical systems and using Lyapunov stability analysis for dissipative systems, allows a deep and unifying understanding of gradient-like and proximal optimization algorithms. Playing with metric aspects allows to grasp in a unifying way objects as different as Lotka-Volterra equation, and interior point methods in convex optimization. Moreover, they offer flexible models for describing real-world decision processes and equilibrium in game theory.

While exploring this rich domain, each of the following topics has been the occasion of discovering new mathematical horizons, new colleagues, and to participate in the training of young mathematicians, all equally enriching experiences!

a) I have a high regard for A. Auslender who has been communicating to me his love for optimization. He helped me discovering several optimizing schools, the Russian, the German, the Brazilian, and last but not the least the Chilean optimizing school.

Indeed, he played a decisive role in the development of the Chilean optimizing school. As a justified reward, he was made in 2005 Doctor Honoris Causa of the University of Chile. On this occasion, the VII French-Latin American Congress on Applied Analysis was organized in Santiago. With R. Cominetti and M. Teboulle we have edited a special issue of Mathematical Programming, Series B, which has been devoted to this event, with a foreword dedicated to A. Auslender. Because of the many contributions, it has been published only in 2009!

I like to remember my first contact with the Chilean optimization community. I was still in Perpignan, and A. Auslender was still in Clermont-Ferrand when he called me, very enthusiast, about his Chilean student R. Cominetti, who, he told me, is exceptional. He was right, R. Cominetti soon became one of the best optimizer-analyst of his generation. I had the great pleasure to belong to his PhD jury in 1989, and further write a couple of papers with him. One concerned the coupling of approximation methods with the steepest descent dynamic in convex optimization.

The other concerned the L^p approximation of variational problems in L^∞ and the corresponding entropy selection property. Since, this subject has known interesting developments with the PhD thesis of T. Champion and D. Torralba, who both visited Santiago. I have been sharing with R. Cominetti, as a co-director, the supervision of the PhD thesis of F. Alvarez. During his one year and a half stay in Montpellier, I have established friendly relationship with F. Alvarez, a remarkable mathematician, and reliable person. We have been writing together several papers concerning mostly second-order dynamics and corresponding algorithmic aspects.

At that time, quite all my PhD students visited CMM in Santiago, which as a distinctive feature was supported by CNRS. J. Bolte, A. Cabot, T. Champion, S. Guillaume, D. Torralba, they all learned much from the high level and challenging atmosphere at CMM, which owed much to the wise and clever direction of R. Correa. Despite F. Auslender and I have not written collaborative paper, we have been writing both with M. Teboulle! Indeed, M. Teboulle visited us twice in Montpellier, and we produced a couple of papers on the algorithmic version of Lotka-Volterra dynamic, putting to the fore the Riemannian structure (possibly singular) underlying interior point methods in convex optimization. Again this has been an enriching experience, M. Teboulle has a communicative enthusiasm, and I have got much from him concerning numerical aspects of optimization.

b) An exciting mathematical adventure started by a remark of R. Cominetti while he visited us in Montpellier in 1998. We knew from J.-J. Moreau that, from a mechanical point of view, the steepest descent method can be viewed as a drop of water sliding along the graph of the function. Indeed, R. Cominetti wondered whether one could consider instead a "heavy ball with friction". Intuitively, one can easily conceive that the ball, after some possible oscillations, will ultimately stabilize at a local minimum of the function. The first step of this program was realized a couple of years later, in a seminal paper I wrote in collaboration with X. Goudou and P. Redont. This paper was precisely called "the heavy ball with friction method". It appeared in 2000 in the first volume of Communications in Contemporary Mathematics, a new journal that H. Brézis was launching. Astonishingly, we discovered a couple of years later, without any connections between us, that the same terminology had already been used by B. Polyak, when studying an algorithm related to the same dynamics. Definitively the terminology was adopted! We have been progressively discovering how rich this dynamical approach turns out to be, its strong connections with dissipative dynamical systems and PDE's, asymptotic stabilization in control theory, unilateral mechanics, and decision sciences. At the same period, F. Alvarez published in SIAM J. Control Optim. a fundamental paper establishing the convergence properties of the trajectories in the case of a convex potential.

Guided by the analogy with the steepest descent analysis, I was tempted to consider the case of a nonsmooth convex potential. In line with the seminal work of M. Schatzman, this naturally led us to consider corresponding second-order differential inclusions, with applications to unilateral mechanics, and shocks laws. Of course, one needs to consider speed as a BV function, and acceleration as a vectorial measure. An other mathematical breakthrough has been accomplished in 2002, in a joint paper with F. Alvarez, J. Bolte and P. Redont, published in J. Math Pures et Appl., where we

considered a second-order gradient-like dissipative system with Hessian-driven damping. As a striking property, this system permits to model non-elastic shock laws with restitution coefficient, in the line with the work of L. Paoli and M. Schatzman.

More recently, together with P.E. Mainge, we had the opportunity to study the effect of the introduction of non-potential terms in such dynamics. P.E. Mainge is a young researcher, he got his HDR thesis under the supervision of A. Moudafi, who is one of my former student and who is professor at the Université des Antilles-Guyanes in Martinique. In some sense, P.E. Mainge is my grandson! I had the pleasure to belong to his HDR jury in Fort de France in 2008, and discover a talentuous promising mathematician.

c) Another important encounter was in Barcelona in 2003. During a meeting organized by E. Martinez-Legaz, I had the opportunity to talk with A. Soubeyran, professor in economics at GREQAM, Aix-Marseille. While explaining him my recent discoveries about dynamical systems with inertia, we realized together the importance of being able to model inertia in decision sciences and economics. We first had in mind a better understanding of the mechanism which allows an agent to pass from a temporary routine to another one. From this, we have been starting an active and friendly collaboration, with papers covering a large spectrum from behavioral psychology to numerical algorithms!

As a basic concept, we have been led to put to the fore the notion of cost-to-move (also called cost-to-change). The cost-to-move function can be viewed as a metric, quasi-distance or relative entropy, which measures the difficulty for the agent to move in the decision or performance space. So doing, the decision space become structured, being equipped with a cost-to-move function. Using this framework, we introduced the "Worthwhile-to-Move"incremental principle, which states that admissible moves (changes) for the agent are such that the marginal gain has to be greater or equal than a fraction of the costs to move (the fraction which is not taken into account represents some sacrifices that that agent may accept in order to go further). This represents limited sacrifices during the transition.

Besides its realistic modeling aspects, introduction of costs-to-move in the dynamics confers them remarkable asymptotic convergence properties, and hence desirable algorithmic properties. They play the same role as friction in mechanics, they induce dissipative effects. We have been introducing a classification typology of costs-to-move:

- Low local cost-to-move terms correspond to viscous friction in mechanics. Despite the fact that they asymptotically vanish, they allow to stabilize the system to a critical point of the gain function (a global maximum in the concave case), even in the case of a continuum of equilibria. In optimization, they give rise to proximal algorithms, with a new interpretation of the regularizing term as a cost to move.
- High local cost-to-move terms correspond to dry friction in mechanics. Generically with respect to the initial data, they stabilize the system within finite time. The limiting equilibrium corresponds no more to a critical point of the gain function, but to a solution of the ϵ-variational principle of I. Ekeland (ϵ being linked to the friction parameter).

This model is quite flexible, second-order dynamics naturally appear when introducing cost to change the speed (a joint paper in J. Convex Anal. 2006, for the 65th birthday of J-P. Aubin). More recently, we have been improving the model in an even more realistic way, by introducing an adjoint exploration process, and so define "Local search proximal algorithms."

d) Non-autonomous gradient-like systems play an important role in the asymptotic control and stabilization of nonlinear systems. For example, in the presence of a continuum of equilibria, the introduction of a Tikhonov regularization term in the dynamic, with a time-dependent coefficient which slowly vanishes, allows to asymptotically select an equilibrium whose norm is minimal. We have been developping this program with M.-O. Czarnecki in a couple of papers, which have been published in J. Diff. Equations (2002, 2010). In the first one we considered a nonlinear oscillator with non-isolated equilibria. In the last one, we considered the more general situation where the dynamic is governed by the sum of several gradient operators acting at different time scales, which allows modeling coupled systems with time variable coupling forces, and the emergence of various equilibria.

Still regarding non-autonomous dynamics, B. Svaiter (from IMPA, Rio, Brazil) and I have recently introduced a continuous dynamical system related to Levenberg-Marquard regularization of Newton method, for solving inclusions governed by general maximal monotone operators in Hilbert spaces. This result has been obtained while I visited IMPA, July 2009, at the occasion of the VIII Brazilian workshop on continuous optimization, celebrating A. Iusem 60th birthday. This meeting has been an opportunity for me to better know A. Iusem, a smart mathematician who masters both advanced analysis theories and numerical optimization.

e) Developing these ideas, together with their links with game theory, has been one of the objectives of the ANR program "decisionprox". From 2005 to 2008, I have been responsible for this program which has been realized in collaboration with S. Sorin from the Equipe de Combinatoire et Optimisation, Paris VI. At this occasion, we have been discovering the rich connections between repeated games, the favorite subject of S. Sorin, and the asymptotic properties of various autonomous, and non-autonomous dynamical systems arising in optimization. I have learned much from him about games, which further helped me developing algorithmic results related to best reply dynamics for potentials games.

J. Bolte, one of my former PhD students, who is now "Maître de Conférences" in the laboratory of S. Sorin in Paris VI, has been playing a crucial role in the success of this program. Thanks to his deep knowledge of Lojasiewicz theory, and of its further extension by Kurdyka, we have been able to obtain the first convergence result of proximal algorithms for general classes of nonconvex nonsmooth functions (Math Programming, 2009). The key idea was to mimic the historical proof of Lojasiewicz for analytic functions, and to transpose it in a discrete time setting by introducing corresponding cost-to-move terms. This idea has known a remarkable extension to the alternating proximal minimization of structured nonconvex nonsmooth functions, so providing a general nonconvex extension of the celebrated von Neumann alternating projection algorithm and Gauss-Seidel method (Mathematics of Operations Research, 2010, in collaboration with J. Bolte, P. Redont and A. Soubeyran). It is an occasion

for me to recall the important contribution to this subject and related questions like metric regularity, tame analysis, of A. Ioffe, A. Lewis, both smart mathematicians and persons. The work of A. Lewis and J. Malick (MOR, 2007) on this subject is a valuable reference. I would like also to thank A. Haraux and M.A. Jendoubi who made me first discover the work of Lojasiewicz, and the important role of analyticity in gradient systems.

Even for convex optimization, performing proximal minimization steps provides new insight on alternating minimization and projection algorithms. So doing, one can obtain convergence results under minimal assumptions. As a striking example, in a recent paper with M. Soueycatt (Pacific J. Optimization, 2009), we have been starting from the seminal work of R. Glowinski on primal-dual methods for convex constrained problems. Just assuming that the set of equilibria is nonempty, we have been able to prove the convergence of an alternate proximal minimization/maximization procedure applied to the augmented Lagrangian formulation of the problem. I have been learning much while reading the important contributions to this suject and related questions (like iterates of contractions) of H.H. Bauschke, J.M. Borwein, P.-L. Combettes, J. Eckstein, A. Iusem, A. Lewis, S. Reich, M. Teboulle.

My connection with the Chilean school has been even reinforced when F. Alvarez and S. Sorin invited me to be a reviewer of the PhD thesis of their student J. Peypouquet, who worked on related questions. I discovered a new talentuous friendly mathematician, with whom I have recently produced a couple of papers.

We have been quite proud, when our program has been selected for the presentation of the final results of the ANR Blanc, which took place in February 2009 at the Cité des Sciences et de l'Industrie of Paris. Since then, a new ANR program 0SSDAA has been starting in 2008, including the earlier participants and P.-L. Combettes.

But that is another story, which has been starting remarkably well, with a recent result concerning parallel splitting methods for coupled monotone inclusions, in collaboration with P.-L. Combettes and L. M. Briceno-Arias, and which has been accepted in SIAM Journal on Control and Optimization. L. M. Briceno-Arias is a young promising Chilean student of P.-L. Combettes, ... and so, the story never ends!

I realize that I have been quite long! But there has been so many nice moments to share together.

Montpellier, March 12, 2010 Hedy Attouch