

## Seminario de Grafos

**Speaker:** Taruni Sridhar (CMM)

**Title:** On  $(n, m)$ -chromatic numbers of graphs with bounded sparsity parameters.

**Abstract:** An  $(n, m)$ -graph is characterised with  $n$  types of arcs and  $m$  types of edges.

A homomorphism of an  $(n, m)$ -graph  $G$  to an  $(n, m)$ -graph  $H$ , is a vertex mapping that preserves adjacency, direction, and type. The  $(n, m)$ -chromatic number of  $G$ , denoted by  $\chi_{n,m}(G)$ , is the minimum value of  $|V(H)|$  such that there exists a homomorphism of  $G$  to  $H$ . The theory of homomorphisms of  $(n, m)$ -graphs has connections with graph theoretic concepts like harmonious coloring, nowhere-zero flows; with other mathematical topics like binary predicate logic, Coxeter groups; and has applications to the Query Evaluation Problem (QEP) in graph database.

In this talk, we show that the arboricity of  $G$  is bounded by a function of  $\chi_{n,m}(G)$  but not vice versa. Additionally, we show that the acyclic chromatic number of  $G$  is bounded by a function of  $\chi_{n,m}(G)$ , a result already known in the reverse direction. Furthermore, we prove that the  $(n, m)$ -chromatic number for the family of graphs with maximum average degree less than  $2 + \frac{2}{4(2n+m)-1}$ , including the subfamily of planar graphs with girth at least  $8(2n+m)$ , equals  $2(2n+m) + 1$ . This improves upon previous findings, which proved the  $(n, m)$ -chromatic number for planar graphs with girth at least  $10(2n+m) - 4$  is  $2(2n+m) + 1$ . It is established that the  $(n, m)$ -chromatic number for the family  $\mathcal{T}_2$  of partial 2-trees is bounded below and above by quadratic functions of  $(2n+m)$ , with the lower bound being tight when  $(2n+m) = 2$ . We prove  $14 \leq \chi_{(0,3)}(\mathcal{T}_2) \leq 15$  and  $14 \leq \chi_{(1,1)}(\mathcal{T}_2) \leq 21$  which improves both known lower bounds and the former upper bound. We prove the first theoretical proof for the upper bound in both the cases.

**Lugar/hora:** Viernes 6 de Agosto , 2024 / 10.00-11.00, Sala Jacques L Lions. (7° Piso)