1 Syntax

$$
\begin{array}{lll}
e ::= c \in \mathbb{Z} & e_e ::= \cdot +_1 e & s ::= \mathit{skip} \\
\quad \mid \mathsf{x} \in \mathit{Var} & \quad \mid \cdot +_2 \cdot & \quad \mid s_1; s_2 \\
\quad \mid e_1 + e_2 & & \quad \mid \mathsf{x} := e \\
 & & \quad \mid \mathit{if}\ (e > 0)\ s_1\ s_2 \\
 & & \quad \mid \mathit{while}\ (e > 0)\ s
\end{array}
\qquad
\begin{array}{l}
s_e ::= \mathsf{x} :=_1 \cdot \\
\quad \mid \cdot ;_1 s_2 \\
\quad \mid \mathit{if}_1\ s_1\ s_2 \\
\quad \mid \mathit{while}_1\ (e > 0)\ s \\
\quad \mid \mathit{while}_2\ (e > 0)\ s
\end{array}
$$

2 Semantics

2.1 Expressions

$$\textsc{red-const}(c)\ \frac{}{E, c \Downarrow c} \qquad \textsc{red-var}(\mathsf{x})\ \frac{}{E, \mathsf{x} \Downarrow E[\mathsf{x}]}\ \mathsf{x} \in \mathit{dom}(E) \qquad \textsc{red-var-undef}(\mathsf{x})\ \frac{}{E, \mathsf{x} \Downarrow \mathit{err}}\ \mathsf{x} \notin \mathit{dom}(E)$$

$$\textsc{red-add}(e_1, e_2)\ \frac{E, e_1 \Downarrow r \qquad E, r, \cdot +_1 e_2 \Downarrow r'}{E, e_1 + e_2 \Downarrow r'} \qquad \textsc{red-add-1}(e_2)\ \frac{E, e_2 \Downarrow r \qquad E, v_1, r, \cdot +_2 \cdot \Downarrow r'}{E, v_1, \cdot +_1 e_2 \Downarrow r'}$$

$$\textsc{red-add-2}\ \frac{}{E, v_1, v_2, \cdot +_2 \cdot \Downarrow v_1 + v_2}$$

2.2 Statements

$$\frac{}{E, skip \Downarrow E}\ \textsc{red-skip}$$

$$\frac{E, s_1 \Downarrow r \qquad r, \cdot;_1 s_2 \Downarrow r'}{E, s_1; s_2 \Downarrow r'}\ \textsc{red-seq}(s_1, s_2)$$

$$\frac{E, s_2 \Downarrow r}{E, \cdot;_1 s_2 \Downarrow r}\ \textsc{red-seq-1}(s_2)$$

$$\frac{E, e \Downarrow r \qquad E, r, \mathsf{x} :=_1 \cdot \Downarrow r'}{E, \mathsf{x} := e \Downarrow r'}\ \textsc{red-asn}(\mathsf{x}, e)$$

$$\frac{}{E, v, \mathsf{x} :=_1 \cdot \Downarrow E\,[\mathsf{x} \leftarrow v]}\ \textsc{red-asn-1}(\mathsf{x})$$

$$\frac{E, e \Downarrow r \qquad E, r, if_1\ s_1\ s_2 \Downarrow r'}{E, if\ (e > 0)\ s_1\ s_2 \Downarrow r'}\ \textsc{red-if}(e, s_1, s_2)$$

$$\frac{E, s_1 \Downarrow r}{E, v, if_1\ s_1\ s_2 \Downarrow r}\ v > 0\ \textsc{red-if-1-pos}(s_1, s_2)$$

$$\frac{E, s_2 \Downarrow r}{E, v, if_1\ s_1\ s_2 \Downarrow r}\ v \leqslant 0\ \textsc{red-if-1-neg}(s_1, s_2)$$

$$\frac{E, e \Downarrow r \qquad E, r, while_1\ (e > 0)\ s \Downarrow r'}{E, while\ (e > 0)\ s \Downarrow r'}\ \textsc{red-while}(e, s)$$

$$\frac{}{E, v, while_1\ (e > 0)\ s \Downarrow E}\ v \leqslant 0\ \textsc{red-while-1-neg}(e, s)$$

$$\frac{E, s \Downarrow r \qquad r, while_2\ (e > 0)\ s \Downarrow r'}{E, v, while_1\ (e > 0)\ s \Downarrow r'}\ v > 0\ \textsc{red-while-1-pos}(e, s)$$

$$\frac{E, while\ (e > 0)\ s \Downarrow r}{E, while_2\ (e > 0)\ s \Downarrow r}\ \textsc{red-while-2}(e, s)$$

2.3 Aborting Rules

$$\frac{}{\sigma, e \Downarrow err}\ \mathbf{abort}\,\sigma\ \textsc{red-error-expr}(e)$$

$$\frac{}{\sigma, s \Downarrow err}\ \mathbf{abort}\,\sigma\ \textsc{red-error-stat}(s)$$

$$\frac{\sigma = C\,[err]}{\mathbf{abort}\,\sigma}$$